MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Expanding cone and applications to homogeneous dynamics

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Contains incorrect or incomplete proofs
03	**Novelty**	No non-novelty findings

Statements

Correct The explicit expanding-cone theorem, quantitative nonescape on semisimple quotients, ineffective equidistribution, effective multiple equidistribution, and the ergodic consequences are correct at their stated scope. The proof of Theorem 1.3 contains a finite-index error, but replacing one exponent by a factorial repairs it and consequently preserves Theorem 1.5.

Theorems 1.2 and 1.4: The cone characterization supplies the divergence condition needed for equidistribution

Version of record · Theorem 1.2, Lemmas 2.1–2.9, and Theorem 1.4

Correct

The dual-cone argument identifies A_U^+ with $\exp(\mathfrak{a}_{\mathfrak{u}}^+)$ by testing every nontrivial irreducible representation on its U-fixed weights. For a sequence drifting away from every wall, each nonzero U-fixed weight tends to infinity. That is precisely the $C(\rho \oplus \bar{\rho})$-divergence condition (3.13) in the exact Shah–Weiss proof, so the limiting homogeneous measure in Theorem 1.4 follows. The numerical slips in the worked SL_{m+n} example do not enter this general proof.

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Theorem 1.3: Quantitative nonescape is valid on the closed expanding cone

Version of record · Theorem 1.3 and Section 3, especially Lemmas 3.2, 3.3, and 3.7

Correct after verified repair

For arithmetic factors, polynomial goodness and exterior-power quantitative nondivergence give a uniform power of ε. For rank-one factors, Buenger–Zheng's one-parameter estimate is uniform in the unipotent direction, and polar integration gives the same type of estimate on a U-ball. The lattice decomposition then reduces the general semisimple quotient to these factors. Lemma 3.7 incorrectly raises a coset representative to the subgroup index; using the factorial of that index, as detailed in the proof finding, restores the injectivity-radius comparison. All estimates remain uniform for $a \in \overline{A_U^+}$ because the relevant U-fixed weights are at least one on the closed cone.

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Theorem 1.5: The effective multiple-equidistribution induction closes after the nonescape repair

Version of record · Lemmas 4.1–4.4, Theorem 4.5, and proof of Theorem 1.5

Correct after verified ancestor repairs

The spectral-gap estimate gives exponential matrix-coefficient decay along b_t because b_1 has a nontrivial projection to every simple factor of H. Thickening an H_b^+-slice produces a mixing error $r^{-3\ell}e^{-t\delta_0}$, while Theorem 1.3 bounds the cusp part by r^{δ_1}. The choice $r = \exp(-t\delta_0/(3\ell + \delta_1))$ balances the two. Iterating Theorem 4.5 over the cumulative parameters yields (1.10), since every consecutive product has depth at least $\lfloor \mathbf{a} \rfloor$. The audited defects in Buenger–Zheng and Kleinbock–Margulis have verified repairs and do not invalidate the exact imported statements used here.

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Theorem 1.6, Corollary 1.7, and Theorem A.1: The periodic and pointwise consequences follow from the effective estimate

Version of record · equations (4.19), (A.1)–(A.4), and their proofs

Correct

For a periodic U-orbit, translating a fundamental domain removes the smooth-density comparison that fails outside the fully expanding cone, leaving the same thickening and nonescape bounds. In the appendix, Theorem 4.5 applied to the $2m$ time parameters gives exponential two-time correlation decay after centering the last observable. The cited almost-sure criterion then yields the stated $T^{-1/2} \log^{3/2+\varepsilon} T$ error scale, and the induction over the observables is valid in the version of record.

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Proofs

Contains incorrect or incomplete proofs The central analytic arguments are complete, but Lemma 3.7 uses a false finite-index power assertion in the step that transfers injectivity radius from a finite-cover quotient. Replacing the subgroup index by its factorial repairs the argument. The remaining defects are local formula, notation, or bibliographic errors.

Lemma 3.7: A subgroup of index n need not contain every nth power

Version of record · Section 3.3, Lemma 3.7 · paragraph beginning with the cardinality n

Incomplete as written · verified repair

Let $D = \rho(\Gamma)$ and $E = D \cap \Gamma_0$, with $[D : E] = n$. The proof asserts $\rho(\gamma)^n \in E$ for every $\gamma \in \Gamma$. This is false when E is not normal: for example, an element can act as a transposition on a three-element coset space. Set $N = n!$ instead. The action of D on D/E maps every element to a permutation of n letters, so d^N lies in the kernel of that action and hence in E for every $d \in D$. Replacing n by N in the radius bounds and in the bounded-torsion neighborhood gives $(g_1^{-1} g_2)^N = 1$ and then $g_1 = g_2$, exactly as required. This change affects constants only and proves the claimed injectivity comparison.

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Section 2.3: The worked SL_{m+n} root data contain two numerical slips

Version of record · Section 2.3, paragraph after the display defining Φ, Φ^+, and Π

Typos

For the upper-right $m \times n$ block unipotent radical, the omitted simple root is $e_m - e_{m+1}$, not $e_m - e_{m-1}$. Also, with the Killing form $B(X, Y) = 2(m+n)\,\mathrm{tr}(XY)$ on $\mathfrak{sl}_{m+n}$, the diagonal entries of $s_{e_i - e_j}$ are $1/(2(m+n))$ and $-1/(2(m+n))$, not $1/(m+n)$ and $-1/(m+n)$. The following display correctly lists $\Phi(\mathfrak{u}) = \{e_i - e_{m+j}\}$, and multiplying every s_α by the same positive scalar does not change the cone, so Example 1.1 and Theorem 1.2 remain correct.

Source: IMRN version of record

Theorems 1.3 and 1.5 source chain: Unavailable publisher files do not leave the imported claims unsupported

Version of record · Sections 3–4 · citations [2], [7], [11], [16], [18], [20], [23], and [25]

Verified with exact-form caveats

The Cambridge version of record for Buenger–Zheng was not publicly downloadable, but its sole arXiv proof-bearing form was audited in full; the marked rank-one Theorem 1.1 is valid after the explicit constant correction and nilpotent-hull repair recorded there. That fallback's unsupported product-only Theorem 1.5 is not used by Shi. The public typeset Kleinbock–Margulis article was audited, and its unrelated return-time gap does not affect Lemma 2.4.7 used for smoothing. The final public forms of the quantitative-nondivergence and EMV inputs were also audited. The exact older Margulis–Tomanov publisher file was not confirmed open, but its big-cell decomposition used in Lemma 4.1 follows independently from the stable, central, and unstable root-space decomposition. Thus no inaccessible non-book ancestor forces an unsupported-statement or unsupported-source finding for Shi's Theorems 1.3 or 1.5.

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Lemma 4.3 and reference [32]: One group symbol and one bibliography record are wrong

Version of record · proof of Lemma 4.3 and bibliography entry [32]

Typos

The mixing line in Lemma 4.3 writes $\langle \varphi, g_{-t}\psi \rangle$ once, although the one-parameter group is denoted b_t everywhere; the required term is $\langle \varphi, b_{-t}\psi \rangle$. Reference [32] identifies the right Shah–Weiss title, authors, theorem number, and equation (3.13), but gives the wrong journal data: the article is in Ergodic Theory and Dynamical Systems 20 (2000), 567–592, not volume 16 (1996), 1–28. The exact public author copy was inspected and confirms the source use, so neither typo changes a proof.

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Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Version of record · International Mathematics Research Notices 2021 (2021), no. 9, 7060–7095 · public full-text HTML

2. IMRN version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT