MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Mittelwerte über Gitter

Wolfgang Schmidt

Reviewed source	Monatshefte für Mathematik version of record · volume 61 (1957), pages 269–276
Exact version	Version of record
Journal citation	Wolfgang Schmidt. Mittelwerte über Gitter. Monatshefte für Mathematik 61 (1957), 269–276.
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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

Statements

Correct The parabolic mean-value identity, the independent-tuple formula, and Schmidt's Rogers expansion are correct at their stated scope. In particular, Satz 2, Satz 3, and Lemma 4 provide a valid replacement for the analytic part of Rogers's mean-value proof for every nonnegative Borel function when $k < n$, with extended value $+\infty$ allowed.

Satz 1: The parabolic one-vector mean-value formula and deformation consequence are correct

Printed pages 270–273 · equations (4)–(7) and Satz 1

Correct

Averaging first over the compact unipotent quotient and then over the lower-right SL_r quotient reduces the sum over integer vectors with nonzero last-r projection to the ordinary Siegel integral. The product measure is normalized in equation (6), and nonnegativity permits every exchange of sum and integral. The stated deformation consequence uses the case $r = 2$ and a determinant-one diagonal scaling; the vectors in the exceptional coordinate plane leave the fixed compact support, after which the average supplies the required lattice.

Source: Schmidt 1957 version of record, printed pages 270–273

Satz 2: The independent-tuple identity holds for every $m < r \leq n$

Printed pages 273–274 · Satz 2 and equations (8)–(10)

Correct

For H_m^r, the set of integer m-tuples whose last-r projections are independent, an integral unimodular change of coordinates places the first $m-1$ projections in a coordinate subspace. The admissible last vector is then exactly a vector outside the complementary coordinate plane. Satz 1 evaluates that inner sum, and induction on m yields

$$\int_{F(\mathfrak{D}/\mathfrak{Z})} \sum_{(g_1,\ldots,g_m)\in H_m^r} f(Dg_1,\ldots,Dg_m)\, d\omega_D = \int f(X_1,\ldots,X_m)\, dX_1\cdots dX_m.$$

Taking $r = n$ gives the full independent-tuple formula for $m < n$.

Source: Schmidt 1957 version of record, printed pages 273–274

Satz 3: The Rogers expansion is valid for arbitrary nonnegative Borel input when $k < n$

Printed pages 274–276 · Satz 3, Lemma 3, and Lemma 4

Correct

Rogers's combinatorial Lemma 3 partitions every integer k-tuple uniquely by its rank, a division of the column indices, a denominator q, and an integral matrix G. Schmidt averages each rank-m stratum using Satz 2 and Lemma 4. Because all terms are nonnegative, Tonelli's theorem justifies the countable rearrangement without a separate convergence hypothesis, so equality remains meaningful when both sides are infinite. This proves the advertised formula independently of Rogers's defective fundamental-domain transfer step.

Source: Schmidt 1957 version of record, printed pages 274–276

Scope boundary: The restrictions to nonnegative functions and $k < n$ are explicit and essential to this theorem

Printed pages 269 and 275 · introduction and Satz 3

Correctly scoped

The paper does not assert the formula for signed input without absolute-integrability hypotheses, nor for $k \geq n$. Its proof uses nonnegativity for Tonelli and Satz 2 with rank $m < n$. For bounded compactly supported input, the 1957 paper only points to the forthcoming convergence proof; Schmidt's 1958 paper supplies that finiteness result for the full valid range $k < n$. None of these boundaries weakens the extended-value Borel identity actually stated here.

Source: Schmidt, On the convergence of mean values over lattices (1958), Theorem 2

Proofs

Correct The proofs close at their stated scope. The crucial replacement is genuinely independent at the analytic level: Schmidt proves the independent-tuple mean value through parabolic quotients and uses Rogers only for a finite-dimensional combinatorial decomposition. No step imports Rogers's invalid fundamental-domain invariance argument.

Borel extension: The base Siegel identity is available for arbitrary nonnegative Borel functions

Printed page 270 · discussion following equation (4)

Correct via the cited approximation argument

Schmidt starts from Siegel's formula for bounded compactly supported Riemann-integrable functions, records the bounded-Borel approximation, and then passes to arbitrary nonnegative Borel functions by truncation, allowing divergence. The local exposition is abbreviated but explicitly points to the detailed approximation on Rogers's printed pages 270–273. That cited measure-theoretic step requires only the already valid Siegel identity for the approximants; it does not use the faulty transfer argument in Rogers's proof of his higher-rank formula.

Source: Rogers 1955 version of record, printed pages 270–275 · Borel extension of Theorem 3

Proof of Satz 2: The induction and quotient disintegration have the required invariance

Printed pages 273–274 · equations (9)–(10)

Correct and complete

The integer normal-form matrix W belongs to the discrete subgroup, so replacing D by DW preserves the quotient integral. The nested parabolic subgroup used for the last vector has normalized quotient mass one, equation (6) supplies the disintegration, and Satz 1 applies to the remaining nonzero projection. Repetition removes one vector at a time and terminates at the stated Euclidean integral. The hypothesis $r > m$ guarantees that the complementary projection dimension is positive at every stage.

Source: Schmidt 1957 version of record, printed pages 273–274

Lemma 4: The lattice-index and Jacobian computation give the printed coefficient

Printed page 276 · Lemma 4

Correct and complete

The congruence solutions form a sublattice $\Lambda \subset \mathbb{Z}^m$ with index, hence determinant, $q^m/N(G,q)$. Writing $\Lambda = Q\mathbb{Z}^m$ converts the constrained independent tuples bijectively into unconstrained independent tuples. The same m-variable linear change acts in each of the n coordinate rows, so its Jacobian is $|\det Q|^{-n} = (N(G,q)/q^m)^n$, exactly the factor in Lemma 4 and Satz 3.

Source: Schmidt 1957 version of record, printed page 276

Independence of the repair: The only imported Rogers result is the correct combinatorial rank decomposition

Printed pages 275–276 · Lemma 3 and its use in Satz 3

Correct dependency boundary

Schmidt cites Rogers's equations (36)–(37) for the one-to-one decomposition of a lattice k-tuple into rank data. Rogers proves that lemma algebraically by choosing the first independent columns, clearing the uniquely determined rational coefficients, and checking the inverse correspondence. The lemma is separate from the later integration step. Schmidt's Satz 2 and Lemma 4 replace that integration step, so the proof does not inherit the invalid right-translate argument identified in Rogers's Theorem 1 proof.

Source: Rogers 1955 version of record, printed pages 275–278 · equations (36)–(42)

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Monatshefte für Mathematik version of record · volume 61 (1957), pages 269–276
2. Schmidt 1957 version of record, printed page 276
3. Schmidt, On the convergence of mean values over lattices (1958), Theorem 2
4. Rogers 1955 version of record, printed pages 275–278 · equations (36)–(42)

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT