MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

On badly approximable numbers and certain games

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	**Correct**
02	**Proofs**	**Correct**
03	**Novelty**	**No non-novelty findings**

Statements

Correct The permanence results for winning sets, the winning property and winning dimension of badly approximable numbers, the digit-expansion examples, the Hausdorff-dimension bounds, and the positional-strategy theorem are correct. One mechanical contraction-parameter typo in the proof of the countable-intersection theorem does not alter its statement.

Theorems 1–3: Local-isometry invariance, countable intersections, and badly approximable numbers

Journal pages 184–189 · Theorems 1–3 and their proofs · version of record

Correct

The local-isometry argument transfers a winning strategy after the required strict parameter shrinkage $\alpha' < \alpha$ with $\alpha'\beta' = \alpha\beta$. The scheduled-subsequence construction proves that countable intersections of α-winning sets remain α-winning after the contraction parameter is corrected as recorded under Proofs. The rational-avoidance strategy proves that badly approximable numbers are (α, β)-winning whenever $2\alpha < 1 + \alpha\beta$, and the obstruction for $\alpha > 1/2$ gives winning dimension exactly $1/2$.

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Theorems 4 and 5: The base-expansion winning-dimension examples

Journal pages 189–194 · Theorems 4–5 and proofs · version of record

Correct

The scale-window strategy excludes a prescribed digit at infinitely many positions and therefore establishes the stated winning property for numbers that are not normal to a fixed base. The complementary construction for numbers with infinitely many zero digits supplies both the winning lower bound and an explicit Black strategy above $\alpha_g = ((g-1)^2 + 1)^{-1}$, proving the exact winning dimension.

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Theorem 6: The branching condition yields the stated Hausdorff-dimension lower bound

Journal pages 194–197 · Theorem 6, Lemmas 20–22, and proof · version of record

Correct

The hypotheses create an m-ary tree of legal strategy chains whose level-k balls have radius $(\alpha\beta)^{kt}$ and pairwise disjoint interiors. Lemma 20 bounds by two the number of level balls met by a sufficiently small covering ball. Mapping branches to base-m expansions then gives Hausdorff dimension at least $\log m/(t|\log(\alpha\beta)|)$. The Euclidean and Hilbert-space corollaries follow from the available branching counts.

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Theorem 7: Every winning set admits a positional winning strategy

Journal pages 198–199 · Theorem 7 and proof · version of record

Correct

The well-order construction assigns to each current move a canonical compatible history. If the construction reaches a terminal alternative, the current ball is already contained in the target; otherwise the canonical prefixes extracted from any purported losing play form an infinite chain for the original winning strategy. In both cases the positional strategy forces the intersection into the target set.

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Proofs

Correct The central proofs are correct and complete after one uniquely determined typographical correction in the effective contraction parameter used to interleave countably many strategies.

Theorem 1: Uniform local distortion estimates transfer every legal selected move

Journal pages 184–186 · Lemma 10 and proof of Theorem 1 · version of record

Correct and complete

Compactness supplies uniform continuity and positive lower bounds for the local metric multiplier. After entering a sufficiently small ball, the two inclusions in Lemma 10 make the pulled-back and pushed-forward balls legal with parameters $\alpha' = \alpha(1-\varepsilon)^2$ and $\beta' = \beta(1-\varepsilon)^{-2}$. Radii tend to zero, so the unique outcome in the target space is the image of the outcome governed by the original winning strategy.

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Proof of Theorem 2: The induced Black contraction begins with β, not α

Journal page 187 · proof of Theorem 2, three occurrences of the induced parameter · version of record

Typo

For the strategy assigned to S_ℓ, successive selected Black moves are 2^ℓ rounds apart. Relative to the preceding selected White move, the actual contraction is $\beta(\alpha\beta)^{2^\ell-1}$. The page instead prints $\alpha(\beta\alpha)^{2^\ell-1}$, including the special cases $\alpha\beta\alpha$ and $\alpha(\beta\alpha)^3$. Replace the initial α by β in all three occurrences. Each S_ℓ is α-winning for every Black parameter, so the corrected parameter supplies the required strategy; the displayed subsequence is then a legal play and its outcome lies in every S_ℓ. Repair classification: Verified repair. The theorem and its corollary are unchanged.

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Proof of Theorem 3: The rational-separation induction checks all denominator blocks

Journal pages 188–189 · Lemma 15 and proof of the Proposition · version of record

Correct and complete

The choice $\alpha\beta\gamma/2 \le (\alpha\beta)^t < \gamma/2$, where $\gamma = 1 + \alpha\beta - 2\alpha$, synchronizes the game scale with denominator blocks. Distinct reduced fractions in one block cannot both be dangerous in the current ball, and Lemma 15 moves the next selected ball past the single possible danger interval. Induction over the blocks yields a uniform lower bound $|x - p/q| > \delta q^{-2}$ for every rational p/q.

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Proofs of Theorems 4–6: The digit-window and branching constructions preserve the required scales

Journal pages 189–197 · Sections 8–12 · version of record

Correct and complete

For Theorems 4–5, the selected digit positions are separated far enough that only one forbidden cylinder meets the current interval, while the losing strategy repeats a fixed block containing no zero. For Theorem 6, Lemmas 20–22 provide the separation and tree structure needed for the covering estimate. The endpoint choices, radius multipliers, and dimension exponents agree throughout the constructions.

Source: Transactions version-of-record scan

Proof of Theorem 7: The canonical-history construction exhausts all positional plays

Journal pages 198–199 · proof of Theorem 7 · version of record

Correct and complete

The well-ordering selects compatible least predecessors. The finite-chain compactness step constructs an infinite original-strategy chain whenever canonical prefixes continue indefinitely. Applied to a play of the positional strategy, either a current ball is already contained in the target or the extracted chain has target-contained intersection; both alternatives prove the required winning property.

Source: Transactions version-of-record scan

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Transactions version-of-record scan

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT