MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Mean values over the space of lattices

C. A. Rogers

Reviewed source	Acta Mathematica version of record · volume 94 (1955), pages 249–287
Exact version	Version of record
Journal citation	C. A. Rogers. Mean values over the space of lattices. Acta Mathematica 94 (1955), 249–287.
DOI	10.1007/BF02392493
Audit date	August 23, 2026
Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	**Contains wrong statements**
02	**Proofs**	**Contains incorrect or incomplete proofs**
03	**Novelty**	**No non-novelty findings**

Statements

Contains wrong statements The mean-value formulas are correct in their valid range, and the primitive first/second moments needed by the focal dependence graph are correct in every dimension at least three. Theorem 5 is nevertheless false as stated because it includes dimension two. Theorem 6 remains true in dimension two by Schmidt's later independent argument.

Theorem 5: The primitive two-point formula is false when $n = 2$

Printed page 252 · Theorem 5 · equation (12)

Incorrect in the stated dimension-two case

The theorem claims equation (12) for every $n \geq 2$. In dimension two, choose a nonnegative Borel function ρ supported in a sufficiently small neighborhood of $(e_1, \frac{1}{2}e_2)$, with support disjoint from the loci $X_2 = \pm X_1$ and with $\det(X_1, X_2)$ bounded away from every integer. For any unimodular lattice $\Lambda \subset \mathbb{R}^2$, the determinant of any two lattice vectors is an integer, so the entire lattice sum on the left of (12) is zero. The double integral on the right is positive and the two diagonal integrals vanish, a contradiction. The valid local repair is to replace $n \geq 2$ by $n > 2$, exactly as recorded by Schmidt in 1958.

Source: W. M. Schmidt, On the convergence of mean values over lattices, footnote 2

Theorem 6: The metric finiteness/divergence conclusion is correct in all stated dimensions

Printed pages 253 and 283–287 · Theorem 6

Correct · dimension-two proof replaced

For $n > 2$, the first-moment and primitive second-moment bounds yield the stated almost-everywhere convergence/divergence dichotomy. The printed dimension-two divergence proof is not valid because it invokes the false dimension-two case of Theorem 5, but Schmidt's Theorem 2 supplies a stronger discrepancy estimate in $\mathbb{R}^2$ and its corollary gives infinitely many primitive points for every nested Borel family of infinite volume. Thus the conclusion is correct although its printed proof is not.

Source: W. M. Schmidt, A metrical theorem in geometry of numbers, Theorem 2 and Corollary

Theorem 5 for $n \geq 3$: The focal primitive first- and second-moment specialization is correct

Printed pages 251–252 and 279–282 · Theorems 4–5

Correct in the focal range

Let $D \geq 3$ and let N_r count primitive vectors of a unimodular lattice in the symmetric cube $(-r, r)^D$. The primitive Siegel formula and Theorem 5 give

$$\mathbb{E}N_r = \frac{2^D r^D}{\zeta(D)}, \qquad \mathbb{E}N_r^2 = (\mathbb{E}N_r)^2 + 2\mathbb{E}N_r.$$

Since N_r is even, the first and second moments imply

$$\mu\{\delta(\Lambda) < r\} = \frac{2^{D-1}}{\zeta(D)} r^D + O_D(r^{2D}).$$

Subtracting the same estimate at r/e gives the fixed-ratio cusp-annulus bound required by the BFK graph. The 1955 proof of the moment formula still inherits the invalid page-256 transfer step through Theorem 3, but Schmidt's independent 1957 Satz 2–3 and Lemma 4 prove the full arbitrary-Borel formula for $k < D$. Thus the focal $D = m+n \geq 3$ cube specialization is valid despite the adverse whole-paper proof status.

Source: W. Schmidt, Mittelwerte über Gitter, Satz 2–3 and Lemma 4

Proofs

Contains incorrect or incomplete proofs Two substantive defects occur. The fundamental-domain argument on printed page 256 uses a false invariance step; later proofs of the Rogers formula replace it. The proof of Theorem 5 applies Theorem 3 outside its range when $n = 2$, producing the false statement above and invalidating the printed dimension-two branch of Theorem 6.

Theorem 1 transfer step: The asserted right-translate decomposition of the fundamental domain is invalid

Printed pages 255–258 · especially page 256 and equation (18)

Incorrect as written · verified replacement proofs

After changing variables, the proof asserts that F^*O^{-1} can be cut into finitely many pieces and returned to F^* by right multiplication with integral unimodular matrices for an arbitrary determinant-one matrix O. That is not a property of a right fundamental domain: right translation conjugates the lattice subgroup. Kim gives an explicit failure using a compactly supported function away from the cusp and $O = \mathrm{diag}(a, a^{-1})$ for large a. This breaks the printed derivation of (18), on which the subsequent transfer to invariant measure rests. Repair classification: Verified repair. Schmidt supplied an alternative proof of Rogers' theorem, and Kim's Hecke-operator argument supplies a later direct replacement for the Rogers integral formula.

Source: Seungki Kim, Mean value formulas on sublattices and flags of the random lattice, arXiv:2005.10874v3, Section 2

Proof of Theorem 5: The dimension-two case invokes Theorem 3 outside its hypotheses

Printed pages 279–282 · equations (49)–(53)

Incorrect as written · statement repaired only by restricting the dimension

The dominated-convergence step (49) and the evaluation leading to (53) invoke Theorem 3 for two linearly independent vectors. Theorem 3 assumes the number of vectors m satisfies $m \leq n - 1$, so it is unavailable when $m = n = 2$. This is not a missing routine endpoint: in dimension two the resulting equation (12) is false by the determinant-support counterexample in the Statements dimension. Repair classification: Verified scope repair $n > 2$; there is no repair preserving the printed $n = 2$ formula. For $D \geq 3$ this endpoint problem disappears, although Schmidt's independent proof is still needed to replace the separate transfer defect inherited by Theorem 3.

Source: Rogers version of record and Schmidt's independent 1957 repair

Proof of Theorem 6 in dimension two: The printed second-moment argument inherits the false endpoint

Printed pages 285–287 · divergent-integral branch of Theorem 6

Incorrect as written · verified independent repair

The displayed variance identity is obtained by applying Theorem 5 to the truncated function $h o_R$. For $n = 2$ that input is false, so the contradiction argument does not prove the stated conclusion. Repair classification: Verified repair. Schmidt's separate planar determinant-integral argument proves a quantitative primitive discrepancy bound and hence the dimension-two conclusion of Theorem 6 without using equation (12).

Source: W. M. Schmidt, A metrical theorem in geometry of numbers, Theorem 2

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Acta Mathematica version of record · volume 94 (1955), pages 249–287
2. W. M. Schmidt, On the convergence of mean values over lattices, footnote 2
3. W. M. Schmidt, A metrical theorem in geometry of numbers, Theorem 2
4. Rogers version of record and Schmidt's independent 1957 repair
5. Seungki Kim, Mean value formulas on sublattices and flags of the random lattice, arXiv:2005.10874v3, Section 2

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT