MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Khintchine's singular Diophantine systems and their applications

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Contains unsupported statements
02	**Proofs**	Contains incorrect or incomplete proofs
03	**Novelty**	No non-novelty findings

Statements

Contains unsupported statements The article explicitly presents itself as a survey and properly attributes most reported results. Its proved extension theorem and exponent refinements are supported after local notation corrections. The arbitrary-dimensional construction in Theorem 68 is not verified because the published text gives only a sketch for dimension three and leaves its key ambient exclusion step unproved.

Surveyed results: Cited theorems are distinguished from present-paper claims

Russian Mathematical Surveys 65 (2010), pp. 433-496 and 499-511, attributed historical and recent results

Correctly treated as cited results

The abstract and introduction identify the work as a survey. Classical and modern results are normally accompanied by an author's name, a bibliographic reference, or explicit wording that the result was announced elsewhere. The absence of reproduced proofs for those attributed statements is therefore not treated as a defect in the paper's own arguments.

Theorem 12: Almost every extension preserves the eventual best-approximation sequence

Russian Mathematical Surveys 65 (2010), pp. 450-452, Theorem 12 and equations (37)-(41)

Correct

For each competitor with a nonzero added-coordinate block, the exceptional parameters form a product of n slabs. Summing their measures over the original coordinates, added coordinates, and bounded integer shifts gives the series in (37). Borel-Cantelli then excludes all but finitely many competitors for almost every extension. Competitors with zero added block are controlled by the original best-approximation sequence.

Section 5 exponent refinements: The determinant and rational-plane arguments support the new bounds

Russian Mathematical Surveys 65 (2010), pp. 461-468, Theorems 20, 22, and 24 and their proofs

Correct

In the cases $(m, n) = (1, 3), (2, 2), (3, 1)$, the proofs select maximal blocks of best approximations in a rational two-plane. A nonzero four-vector determinant gives one growth alternative, while the rank-two lattice covolume gives the complementary alternative. Substitution of a power majorant and optimization of the auxiliary growth exponent yields the displayed functions g_1, g_3, and g_2.

Theorem 68: The arbitrary-gap singular-system construction is not fully proved

Russian Mathematical Surveys 65 (2010), pp. 497-499, Theorem 68, Lemma 8, and the proof sketch

Not able to verify

The theorem quantifies over every $m \geq 3$, every prescribed decreasing function ψ, and every increasing sequence $\tau(\nu)$. The text explicitly sketches only $m = 3$. Even there, Lemma 8 asserts without a quantitative argument that sufficiently small sector parameters make the selected relative best approximations exactly all ambient best approximations. The general case $m > 3$ is not addressed, and no complete repair was verified.

Proofs

Contains incorrect or incomplete proofs The Borel-Cantelli argument and the three exponent-refinement proofs close after uniquely determined notation repairs. The construction supporting Theorem 68 remains incomplete both in its dimension-three extension lemma and in its passage to arbitrary dimension.

Proof of Theorem 12: The exceptional-set sum matches the convergence hypothesis

Russian Mathematical Surveys 65 (2010), pp. 451-452, equations (39)-(41)

Correct and complete

After separating the zero added-coordinate block, a fixed competitor contributes $O(\zeta_\nu^n/\|x_{m+1:m^*}\|^n)$ in parameter measure. Counting original coordinates and integer shifts, then summing the added block by maximum norm, produces the power $M_{\nu+1}^{\max(m+n,m^*)}$ and exactly one logarithm in the equality case. This is the summand in (37).

Theorem 12 notation: The exceptional-set setup contains mechanically repairable symbol slips

Russian Mathematical Surveys 65 (2010), pp. 451-452, equations (39)-(41)

Typo

The competitor vector in the line following (39) must use generic y_1 rather than $y_{1,\nu}$, and the exclusion must cover the adjacent lifted best approximations used immediately below. Most decisively, $\Omega_\nu(x, y)$ must require membership in every interval J_ν, not non-membership: it is the bad set where all errors are small, and only that set has the following $O(\zeta_\nu^n)$ measure estimate. The stated goal and the estimate uniquely determine these local corrections; the mathematical argument is unchanged.

Theorems 20, 22, and 24: The determinant and covolume case splits cover all required alternatives

Russian Mathematical Surveys 65 (2010), pp. 461-468, proofs of Theorems 20, 22, and 24

Correct and complete

Each proof partitions the growth of consecutive best-approximation norms into complementary ranges. A nonzero integral determinant supplies the first estimate and the constant covolume of the rank-two lattice across the intervening block supplies the other. The monotone majorants can be chosen as stated, and the optimization equations for g_1, g_3, and g_2 yield the displayed lower bounds.

Section 5 notation: Two published formulas retain uniquely identifiable dimension and function-name slips

Russian Mathematical Surveys 65 (2010), pp. 466-467, proof of Theorem 22 and cases 2-3 in the proof of Theorem 24

Typo

On p. 466 the maximum in the empty-parallelepiped display must run over the two forms, not three. On p. 467 the exponents in cases 2 and 3 use an undefined $g(\alpha(\Theta))$; the function defined on the preceding page and used in the closing polynomial identity is $g_2(\alpha(\Theta))$. Each repair is forced by the dimension or the immediately adjacent definition and does not alter the proof.

Lemma 8: Ambient best-approximation exclusion is only asserted

Russian Mathematical Surveys 65 (2010), pp. 497-498, Lemma 8 and its proof sketch

Incomplete as written - no verified repair supplied

The sketch chooses z_{c+1} in a neighboring lattice layer and $z_{c+2}, \ldots, z_d$ as best approximations relative to a rational two-plane, then says small η_* and δ_* make these exactly the first d ambient best approximations throughout the new sector. That requires uniform separation from every competing point in all other lattice layers and preservation of the prescribed ordering. Neither a quantitative separation bound nor a finite-competitor reduction is supplied.

Proof of Theorem 68: Only the case of three variables is discussed

Russian Mathematical Surveys 65 (2010), pp. 497-499, proof following Theorem 68

Incomplete as written - no verified repair supplied

The proof explicitly restricts its explanation to $m = 3$, and its sectors, rational hyperplanes, and enumeration argument all live in $\mathbb{R}^3$ or $\mathbb{R}^4$. No reduction from general $m \geq 3$ to this case and no higher-dimensional version of Lemma 8 is supplied. The omitted passage is a substantive proof obligation.

Theorem 69: The following independence theorem is explicitly external

Russian Mathematical Surveys 65 (2010), p. 499, paragraph preceding Theorem 69

Correctly treated as cited, not as proved here

The text says that Theorem 69 was announced in reference [34]. The survey does not present a proof, so its absence is not counted as a defect in the paper's own arguments.

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Russian Mathematical Surveys 65(3) (2010), 433-511 (English version of record)

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT