

Correctness, proof completeness, and novelty audit

An optimal bound for the ratio between ordinary and uniform exponents of Diophantine approximation

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Scope	Statements, proofs, and novelty
Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.	

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

Statements

Correct The published article's optimal lower bounds for the ratios between ordinary and uniform Diophantine exponents, in both simultaneous and dual approximation, and its realization of every larger admissible ratio are supported by the arguments in the version of record.

Theorem 1, lower bounds: The ratios are bounded by the positive roots G and G^*

Mathematika 66 (2020), pp. 820-821, Theorem 1; proofs on pp. 825-849

Correct

For simultaneous approximation, the selected best-approximation pattern recursively decomposes into overlapping rational subspaces. Schmidt's height inequality and Lemma 4 force a growth alternative whose optimized exponent is the positive root of $R_{n,\lambda}$. The hyperbolic rotation in Section 5 supplies the corresponding dual estimate governed by $R_{n,\omega}^*$.

Theorem 1, realization: Every admissible larger ratio is realized

Mathematika 66 (2020), pp. 849-853, Section 6 and the second part of Theorem 1

Correct

The self-similar Roy systems satisfy the generalized-system axioms and have explicitly computed liminf and limsup data. Choosing the three parameters fixes the prescribed uniform exponent and any ratio C at or above the root bound; Roy's realization theorem then produces points with exactly those exponents.

Proofs

Correct The local determinant estimate, the recursive rational-subspace construction, the dual reduction, and the Roy-system realization argument are mutually consistent and cover the claimed parameter ranges in the version of record.

Lemma 4: The local growth alternative has the required exponent balance

Mathematika 66 (2020), pp. 825-827, Lemma 4 and equations (16)-(24)

Correct and complete

The determinant comparison splits into the two stated cases. Substitution of the uniform error estimate cancels the auxiliary powers exactly, and the positivity hypothesis gives the needed sign in the second case. Both branches yield the announced height growth exponent $g(s, t)$.

Lemmas 5-8: The recursive best-approximation pattern exists in every dimension

Mathematika 66 (2020), pp. 828-840, Sections 3-4, Lemmas 5-8

Correct and complete

The construction selects consecutive independent blocks and joins two lower-dimensional patterns along their common rational subspace. The recursive weights remain positive and their determinant exponents telescope as claimed, leaving precisely the polynomial recurrence that defines $G(n, \alpha)$.

Dual reduction: The hyperbolic rotation preserves the required height estimates

Mathematika 66 (2020), pp. 840-849, Section 5, especially Lemmas 9-10

Correct and complete

The determinant estimates for the rotated lattices have the stated scale factors, which cancel from the Schmidt-height products. The resulting four alternatives reproduce the dual root equation and cover approximation by one linear form.

Roy-system construction: The sharpness construction computes the claimed exact exponents

Mathematika 66 (2020), pp. 849-853, Section 6

Correct and complete

The component ordering, slopes, switch points, and self-similar endpoint matching satisfy the generalized-system conditions. Direct extrema of the first and last components give the requested ordinary and uniform exponents, including the boundary root case.

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. *Mathematika* 66(3) (2020), 818-854 (version of record)

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

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Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

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Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
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1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
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PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.
2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.
3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.
4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.
5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.
6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.
7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.
8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.
9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.
10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.
11. Write every mathematical expression as valid, compilable LaTeX. Use $\dots$ for inline mathematics and $\dots$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.
12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT