MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Euclidean matchings and minimality of hyperplane arrangements

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Exact arXiv version 2 · explicit fallback from the 2021 journal article

Reviewed source	arXiv:1809.02476v2 (explicit fallback)
Exact version	
Journal citation	Davide Lofano and Giovanni Paolini. Euclidean matchings and minimality of hyperplane arrangements. arXiv:1809.02476v2; published in Discrete Mathematics 344 (2021), 112232.
Audit date	August 23, 2026
Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

Statements

Correct The Euclidean matching, acyclicity, properness, minimality, and Betti-number consequences are correct in the checked arXiv and accepted-manuscript forms. Two bibliographic or cross-reference slips have uniquely determined repairs and do not change a mathematical claim.

Theorems 3.2 and 4.10: The Euclidean matching has the claimed discrete-Morse properties

arXiv:1809.02476v2 PDF pages 8–9 and 14–18 · Theorems 3.2 and 4.10

Correct

The chamber-to-projection-face construction defines a matching on the Salvetti complex, and the order argument rules out directed cycles. For locally finite arrangements, the compactness and distance estimates give properness. The finite case therefore yields a finite Morse complex with the announced critical-cell description.

Source: Exact arXiv version 2

Theorem 5.13 and Corollary 5.14: Projection-face codimension counts equal Betti numbers

arXiv:1809.02476v2 PDF pages 19–21 · Theorem 5.13 and Corollary 5.14

Correct

For a finite real arrangement and a generic point, exactly one critical cell is associated with each chamber, in degree equal to the projection-face codimension. The number of critical cells equals the sum of Betti numbers, so equality of total counts forces degreewise minimality. This is the exact input used by the focal projection theorem after the standard Poincaré and characteristic-polynomial change of variables.

Remark 5.4 and bibliography: Two local references need typographical repairs

arXiv:1809.02476v2 PDF pages 17 and 28 · Remark 5.4 and reference [Zas97]

Harmless typos

Remark 5.4 says that the displayed alternating path violates condition (ii), but the preceding definition shows it violates condition (i). The region-count source is printed as a 1997 item although the cited Memoirs volume is Zaslavsky's 1975 monograph. Neither slip changes the proof or the identity used in Corollary 5.14.

Proofs

Correct The whole proof was checked in exact arXiv version 2 and compared with the institutional accepted manuscript. The matching involution, no-cycle order, local-finiteness argument, and minimality count close with the stated hypotheses. No unresolved logical gap was found.

Euclidean matching construction: Matched cells have the required unique coface relation

arXiv:1809.02476v2 PDF pages 7–14 · Sections 3–4

Correct

The preferred chamber determined by the Euclidean projection selects one facet whenever a cell is noncritical. Adding or removing that facet is involutive, and the localization identities show that both cells retain the same preferred data. Thus the pairing is a genuine matching on the face poset.

Acyclicity and properness: The monotone distance data prevent cycles and infinite bounded trajectories

arXiv:1809.02476v2 PDF pages 14–18 · proof of Theorem 4.10 and Section 5.1

Correct

Along any reversed matched edge, the ordered projection data change monotonically; a directed cycle would force all changes to be equal and then contradict the selected facet. Local finiteness leaves only finitely many arrangement faces in each compact metric range, giving the required properness.

Finite minimality argument: The topological count is degreewise, not merely total

arXiv:1809.02476v2 PDF pages 18–21 · Theorems 5.12–5.13 and Corollary 5.14

Correct

Morse inequalities give at least the Betti number in each degree, while Orlik–Solomon and Zaslavsky identify the total Betti number with the number of chambers, which is already the total number of critical cells. Since every degreewise excess is nonnegative and their sum is zero, each excess vanishes. This justifies the coefficient-by-coefficient conclusion.

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. The publisher PDF was not freely retrievable. Exact arXiv version 2 and the institutional accepted manuscript were checked; this report does not claim to audit the inaccessible version of record.

Sources checked

1. Exact arXiv version 2

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT