MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Singularity, weighted uniform approximation, intersections and rates

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Contains wrong statements
02	**Proofs**	Contains incorrect or incomplete proofs
03	**Novelty**	No non-novelty findings

Statements

Contains wrong statements The density conclusion in Theorem 1.5(a) and the principal Diophantine applications are correct after the proof repairs recorded below. Theorem 1.5(b), however, is false under its printed abstract hypotheses because a connected member of the aligned family may be a singleton. The forbidden-set scope of Theorem 3.4 is also false without the word 'closed'. The reversed monotonicity word in Theorems 1.13 and 7.2 is a mechanical typo.

Theorem 1.5(a): The abstract density conclusion is correct

Journal pages 2994 and 2998–2999 · Theorem 1.5(a) and Steps 1–3 of its proof · version of record

Correct

Starting in an arbitrary relatively compact open set, total density selects an aligned resonant set meeting the current neighborhood and escaping the next forbidden closed set. Respect and continuity permit a smaller open set whose closure remains inside the previous one and on which every earlier approximation inequality holds at the next height. The nested compact-closure argument gives a common point, and monotonicity of each approximating function supplies the required inequality for every sufficiently large parameter. All countably many systems and forbidden sets are visited.

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Theorem 1.5(b): Connected singleton resonant sets contradict the claimed uncountability

Journal page 2994 · Theorem 1.5(b); journal page 2999 · Step 4 · version of record

Incorrect

Let $Y = \mathbb{N}$ with the discrete metric, let $\mathcal{L} = \{\{j\} : j \in \mathbb{N}\}$, and let $\mathcal{R} = \varnothing$. For every k, take $X_k = Y$, $\varphi_k = \mathrm{Id}$, indices $s \in \mathbb{N}$, $d_{k,s}(x) = 0$ when $x = s$ and $d_{k,s}(x) = 1$ otherwise, heights $h_{k,s} = s + 1$, and $f_k(t) = 1/2$. The space is locally compact, all distance functions are continuous, $\mathcal{L}$ is totally dense relative to the empty collection, respects it, and is aligned with every system. Every $L \in \mathcal{L}$ is connected. Yet every point of Y is uniform by reusing its zero-distance index, so (1.11) equals the countable set Y, contradicting part (b). Step 4 fails at exactly the assertion that adjoining the countable output set preserves (1.8): when $L = \{y\}$, no aligned singleton meeting L can escape $\{y\}$. Requiring every L to have no isolated points repairs the proof and covers all positive-dimensional families used in Sections 3–7, but the printed abstract statement is false.

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Theorem 3.4: The forbidden analytic submanifolds must be closed

Journal page 3002 · statement of Theorem 3.4 · version of record

Incorrect as written · verified repair

The theorem quantifies over an arbitrary countable collection of proper analytic submanifolds, omitting the closedness hypothesis used by Theorems 1.5 and 3.3. As written, take a countable cover of $M_{m,n}(\mathbb{R})$ by proper open balls: every ball is a proper real-analytic submanifold, their union is the whole space, and the displayed set is empty rather than dense and uncountable. Inserting 'closed' before 'analytic submanifolds' is the verified repair; it is exactly the hypothesis invoked by the preceding theorem and by the proof's appeal to the same argument.

Source: Cambridge version of record

Theorems 1.7–1.12: The weighted, manifold, algebraic, and sumset conclusions survive the abstract correction

Journal pages 2995–2997 and 3002–3011 · Theorems 1.7–1.12 and Sections 3–7 · version of record

Correct

Every aligned set used for these uncountability conclusions has no isolated points: rational matrix fibers have positive dimension when $n > 1$, the manifold slices have dimension at least one, the product-fractal slices are perfect, and the intersections of two hyperplanes in the sumset argument have dimension at least one when $n \geq 3$. Thus the corrected form of Theorem 1.5(b) applies. The weighted and polynomial-system implementation defects recorded in Part 2 have explicit verified repairs that preserve the stated conclusions and every parameter range.

Source: Cambridge version of record

Theorems 1.13 and 7.2: The monotonicity hypothesis should say non-increasing

Journal pages 2997 and 3011 · Theorems 1.13 and 7.2 · version of record

Typo

Both statements call f non-decreasing. Replace 'non-decreasing' by 'non-increasing'. The proof explicitly needs f non-increasing in order that the transferred error function h be non-increasing, and the exponent consequence uses decreasing power functions. The intended correction is uniquely fixed by the proof and changes no later argument.

Source: Cambridge version of record

Proofs

Contains incorrect or incomplete proofs The nested construction proving Theorem 1.5(a), the incidence and analytic-continuation arguments, the sumset construction, and the convex-body transference calculation are correct. Step 4 cannot prove the printed uncountability theorem, Equation (3.4) encodes approximation for the most favorable weight instead of every weight, and the polynomial encoding in Theorem 5.1 omits the constant coefficient from the height. The latter two defects have verified repairs that preserve the applications.

Proof of Theorem 1.5(a): The nested topological construction is correct and complete

Journal pages 2998–2999 · Steps 1–3 of the proof of Theorem 1.5 · version of record

Correct and complete

The closure of each chosen open set is nonempty and lies inside its predecessor, so local compactness supplies a nonempty common intersection. At stage ℓ, the construction records aligned indices for the first ℓ systems, increases the common height threshold, and uses continuity at the previous resonant set to impose the next approximation bound. If $T_\ell \leq T < T_{\ell+1}$, the index from stage ℓ has height at most T and its distance is below $f_k(T_{\ell+1}) \leq f_k(T)$. The forbidden sets are avoided one by one.

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Proof of Theorem 1.5(b): Adjoining the alleged countable output need not preserve total density

Journal page 2999 · Step 4 of the proof of Theorem 1.5 · version of record

Incorrect as written

Step 4 asserts that $\mathcal{R} \cup \{\{y_j\} : j \in \mathbb{N}\}$ still satisfies (1.8). Connectedness alone does not imply this: a connected L may be the singleton $\{y_j\}$, in which case every L' meeting L at that point can likewise be trapped in $\{y_j\}$. The discrete counterexample in Part 1 realizes this obstruction and disproves the printed conclusion. Repair classification: Verified repair after strengthening the hypothesis to require that every L have no isolated points; then L itself meets the current open set and is not contained in any singleton, so (1.8) persists.

Source: Cambridge version of record

Theorem 3.4, Equation (3.4): The infimum over weights does not encode simultaneous weighted approximation

Journal page 3002 · Equation (3.4) and proof of Theorem 3.4 · version of record

Incorrect as written · verified repair

The printed distance is $d_{k,(p,q)}(A) = \inf_{\omega \in W_k} \|Aq - p\|_\alpha$. Hence $d \leq f_k(t)$ controls only the most favorable weight, whereas membership in $\mathrm{UA}_{m,n}(f_k, W_k)$ requires the same (p, q) to satisfy the inequalities for every $\omega \in W_k$. For example, with one residual coordinate equal to 0.01, weights $\alpha = 1$ and $\alpha = 2$, and $f_k(t) = 0.05$, the printed infimum is 0.01 but the second quasi-norm is $0.1 > 0.05$. A verified repair is to use $g_k(t) = \min\{f_k(t), 1/2\}$ and the distance $\widetilde{d}_{k,(p,q)}(A) = \min\{1, \sup_{\omega \in W_k} \|Aq - p\|_\alpha\}$, retaining the printed supremum height. Indeed, if $a_{k,i} = \sup_{\omega \in W_k} \alpha_i$, then on the region $|r_i| < 1$ for every residual coordinate one has $\widetilde{d}(r) = \max_i |r_i|^{1/a_{k,i}}$, while $\widetilde{d}(r) = 1$ once some $|r_i| \geq 1$. Thus (3.3) makes $\widetilde{d}$ finite and continuous, with zero set $Aq = p$; its height is finite by the lower bound on the β_j. The inequalities $\widetilde{d} \leq g_k(t) < 1$ and $h \leq t$ give every weighted inequality required in $\mathrm{UA}_{m,n}(f_k, W_k)$. Applying Theorem 1.5 to these repaired systems therefore produces a dense uncountable subset of the printed target, preserving both conclusions of Theorem 3.4.

Source: Cambridge version of record

Proof of Theorem 5.1: The standard linear-form height omits the polynomial's constant coefficient

Journal pages 3006–3007 · definition preceding Theorem 5.1 and its proof · version of record

Incorrect as written · verified repair

The paper defines $H(P)$ as the maximum of all coefficients of P, but maps x to the nonconstant monomials and then uses the standard linear-form system, whose height is only the norm of the nonconstant coefficient vector q. Its assertion of equivalence therefore omits the constant coefficient p: a relation with $\|q\| \le t$ may have $|p| > t$, so the associated polynomial need not satisfy $H(P) \le t$. Replace the standard height by $h_{p,q} = \max\{|p|, \|q\|\}$ while keeping the same affine-hyperplane distance. The zero sets and alignment used by Theorem 1.5 are unchanged, and for $P(x) = q \cdot \varphi_k(x) - p$ with $q \neq 0$ the repaired bound $h_{p,q} \le t$ is precisely $H(P) \le t$. Theorem 1.5 applied with these repaired heights therefore supplies a dense uncountable subset of the printed target, which is sufficient for Theorems 5.1, 5.2, and the simultaneous k-singularity consequence.

Source: Cambridge version of record

Sections 4, 6, and 7: The manifold, sumset, and transference arguments are otherwise complete

Journal pages 3003–3011 · Sections 4, 6, and 7 · version of record

Correct and complete after the stated monotonicity repair

Local graph coordinates produce aligned rational slices of positive dimension, analytic continuation establishes the respect property, and the translated pair-of-hyperplanes family is totally dense relative to affine hyperplanes when $n \ge 3$. In Lemma 7.1, the covolume identity, section-volume estimates, and Minkowski bounds yield a nonzero dual integer point with the required constants. After replacing 'non-decreasing' by 'non-increasing', the scale $\tau(t)$ increases to infinity, the transferred function h is non-increasing, and Theorem 4.2 gives the claimed column-vector rate.

Source: Cambridge version of record

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Cambridge version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT