

# Correctness, proof completeness, and novelty audit

## Logarithm laws for flows on homogeneous spaces

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|                                                                                                                                |                                                                                                                                                                                                                       |
|--------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Reviewed source                                                                                                                | <a href="#">arXiv:math/9812088v2</a> · explicit fallback for inaccessible version of record                                                                                                                           |
| Exact version                                                                                                                  | <a href="#">arXiv:math/9812088v2</a> (explicit fallback)                                                                                                                                                              |
| Journal citation                                                                                                               | Dmitry Kleinbock, Grigory Margulis. Logarithm laws for flows on homogeneous spaces. <i>Inventiones Mathematicae</i> 138 (1999), no. 3, 451–494. Reviewed form: <a href="#">arXiv:math/9812088v2</a> . August 23, 2026 |
| Audit date                                                                                                                     | Statements, proofs, and novelty                                                                                                                                                                                       |
| Scope                                                                                                                          |                                                                                                                                                                                                                       |
| Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. |                                                                                                                                                                                                                       |

### SUMMARY

|    |            |                                         |
|----|------------|-----------------------------------------|
| 01 | Statements | Correct                                 |
| 02 | Proofs     | Contains incorrect or incomplete proofs |
| 03 | Novelty    | No non-novelty findings                 |

## Statements

**Correct** The logarithm laws and Borel–Cantelli conclusions are correct, but this conclusion relies on the verified replacement argument published in arXiv:1706.08570v4: the proof printed in the reviewed version uses a false smooth-approximation lemma.

### Theorems 1.4, 1.7, and 1.9: Logarithm laws for homogeneous and locally symmetric spaces

Sections 1 and 4–6 · arXiv:math/9812088v2

#### Correct

The later correction proves the smooth approximation bound with  $\|h\|_{2,\ell} \leq C\sqrt{\|h\|_1}$ , establishes a compatible second-moment Borel–Cantelli theorem under exponential mixing, and explicitly recovers the original Theorem 4.3. The geometric reduction in Sections 5–6 then gives the stated logarithm laws without changing their hypotheses or constants.

**Source:** [Correction, version 4](#)

### Theorems 8.1–8.3: Metric Diophantine applications

Section 8 · Theorems 8.1–8.3 · arXiv:math/9812088v2

#### Correct

The lattice-point identities and volume integrals in Sections 7–8 convert the Diophantine inequalities into the same cusp events governed by the repaired Borel–Cantelli theorem. Thus the zero-one and logarithm conclusions follow from a verified repair even though the original analytic input is invalid as printed.

## Proofs

### Contains incorrect or incomplete proofs

The proof is incorrect as written. Lemma 4.2 claims a Sobolev bound linear in the target measure, and the proof of Theorem 4.3 applies Proposition 4.1 using that false input. The authors' later correction supplies the valid square-root bound and a new second-moment argument that recovers the central results. The focal change-of-variables Lemma 8.3 is self-contained and unaffected.

### Lemma 4.2: The asserted smooth-approximation regularity is false

Pages 11–12 · Lemma 4.2 · arXiv:math/9812088v2

#### Incorrect as written · verified repair

The paper defines  $(C, \ell)$ -regularity by  $\|\Upsilon^\ell h\|_2 \leq C\mu(h)$  and claims uniform majorants and minorants of cusp indicators with this bound. This is impossible for the claimed minorants as the target measure  $a$  tends to zero: the positive operator  $\Upsilon = 1 - \sum X_i^2$  satisfies  $\|\Upsilon^\ell h'\|_2 \geq \|h'\|_2$ , while  $0 \leq h' \leq 1_A$  and  $\|h'\|_1 \geq ca$  imply by Cauchy–Schwarz that  $\|h'\|_2 \geq \|h'\|_1/\sqrt{a} \geq c\sqrt{a}$ ; the asserted regularity would instead give at most  $C\|h'\|_1 \leq Ca$ . The correction arXiv:1706.08570v4 identifies the mistaken Young-inequality step and proves the valid bound  $\|h\|_{2,\ell} \leq C\sqrt{\|h\|_1}$  as Theorem 1.1. Repair classification: Verified repair.

Source: [Correction, version 4](#)

### Application of Proposition 4.1 in Theorem 4.3: The printed Borel–Cantelli proof applies a valid proposition through a false input

Pages 11–13 · Proposition 4.1, Lemma 4.2, and Theorem 4.3 · arXiv:math/9812088v2

#### Incorrect as written · verified repair

Proposition 4.1 is correct under its stated linear-regularity hypothesis, but Lemma 4.2 does not provide functions satisfying that hypothesis. With only the valid square-root estimates, the correlation sum displayed in Proposition 4.1's proof cannot be reused in the proof of Theorem 4.3. Repair classification: Verified repair. Theorems 1.2–1.3 of arXiv:1706.08570v4 prove a new variance estimate under the same exponential-mixing and exponentially-divergent-sequence hypotheses and explicitly recover Theorem 4.3; all downstream central results can therefore use that replacement.

Source: [Correction, version 4](#)

### Lemma 8.3: The focal approximation-function change of variables is self-contained

Pages 23–24 · Lemma 8.3 · arXiv:math/9812088v2

#### Correct and complete

Writing  $P(\lambda) = -\log \psi(e^\lambda)$ , the proof intersects the nondecreasing graph  $L = P(\lambda)$  with the decreasing affine line determined by  $t$ , then sets  $r(t) = (L(t) - \lambda(t))/(m + n)$ . This gives the required monotonicity and the identity  $\psi(e^{t-nr(t)}) = e^{-t-mr(t)}$ . The inverse construction uses the strictly increasing  $\lambda(t)$ , and the Stieltjes-integral comparison proves convergence equivalence. No external result is used, so the marked BFK dependency terminates at this node.

# Novelty

No non-novelty findings No non-novelty findings.

## METHOD

### Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. Springer's version of record is subscription-controlled. The author-hosted prepublication PDF is not byte-identical to arXiv v2 and is not identified as the version of record, so neither accessible file can safely be represented as the VOR. This report therefore reuses the existing audit of the fixed arXiv v2 source and does not claim to audit the VOR.

### Sources checked

1. [arXiv:math/9812088v2](#) · explicit fallback for inaccessible version of record
2. Correction, version 4

## Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

### PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

### PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

### PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

#### PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.
2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.
3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.
4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.
5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if  $P(\delta)$  is proved for every sufficiently small  $\delta > 0$  and  $P$  is immediately monotone in the required direction, then the claim that  $P(\delta)$  holds for every  $\delta > 0$  requires no correction; the omitted monotonicity step is exposition rather than an audit finding.
6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for  $t > 0$  and subsequently applied at  $t = 0$  remains a formal scope mismatch unless the endpoint case was separately established before the application.
7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.
8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.
9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.
10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.
11. Write every mathematical expression as valid, compilable LaTeX. Use  $\dots$  for inline mathematics and  $\dots$  for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.
12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

**END OF REPORT**