MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

On effective equidistribution of expanding translates of certain orbits in the space of lattices

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Reviewed source	arXiv:math/0702433v4 · explicit fallback for version of record
Exact version	arXiv:math/0702433v4 (explicit fallback)
Journal citation	Dmitry Y. Kleinbock, G. A. Margulis. On effective equidistribution of expanding translates of certain orbits in the space of lattices. Number Theory, Analysis and Geometry, 385–396 (2012). Reviewed form: arXiv:math/0702433v4.
Audit date	August 23, 2026
Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

Statements

Correct The effective equidistribution theorem for expanding translates of the matrix horosphere is correct. Several local notation and sign defects have uniquely determined repairs and do not change the theorem.

Theorem 1.3: Equidistribution is exponentially effective uniformly away from the walls

Pages 2–3 and Section 4 · main theorem and proof · arXiv:math/0702433v4

Correct

Writing $g_{\mathbf{t}} = g_t g_{\mathbf{u}}$ with both t and every coordinate of $\mathbf{u}$ comparable to $\lfloor \mathbf{t} \rfloor$ separates the argument into nondivergence under $g_{\mathbf{u}}$ and exponential mixing under g_t. Quantitative nondivergence removes an exponentially small exceptional part, while smoothing the remaining horospherical pieces and applying exponential mixing gives another exponential error. Optimizing the smoothing and compactness parameters yields the asserted rate.

Uniformity in the base lattice: The compact-set dependence is handled correctly

Sections 3–4 · Corollary 3.4, Proposition 3.5, and proof of Theorem 1.3 · arXiv:math/0702433v4

Correct

The exterior-power expansion lemma gives a lower bound uniform over base lattices in a fixed compact set. Quantitative nondivergence therefore puts all but an exponentially small set in K_ε. The injectivity radius bound $r(K_\varepsilon) \gg \varepsilon^k$ makes the local thickening legal uniformly, and the resulting Sobolev losses are absorbed by the chosen exponential scale.

Proofs

Correct The nondivergence, injectivity-radius, smoothing, and mixing estimates are correct and complete after uniquely determined local notation and exponent-sign repairs.

Equation (1.2): The final contracting diagonal entry has the wrong sign

Page 1 · definition of the standard diagonal flow · arXiv:math/0702433v4

Typo

The matrix is declared to lie in $\mathrm{SL}_{m+n}(\mathbb{R})$ and to have n contracting entries, but its last printed entry is $e^{t/n}$ rather than $e^{-t/n}$. Replacing the final sign by a minus makes the determinant one and matches every later use of g_t.

Definition of $I_{f,\psi}$: The defining integral uses an undefined time element instead of its group argument

Page 1 · display defining $I_{f,\psi}(g,z)$ · arXiv:math/0702433v4

Typo

The left side is $I_{f,\psi}(g,z)$ for an arbitrary $g \in G$, but the integrand is printed as $\psi(g_t hz)$, leaving t undefined and g unused. The following sentence calls this the g-translate, and every subsequent specialization writes $I_{f,\psi}(g_t,z)$; therefore the definition must read $\psi(ghz)$.

Theorem 2.1: The universal quantifier names an unused time variable

Page 3 · exponential mixing theorem · arXiv:math/0702433v4

Typo

The theorem says 'for any $t \geq 0$' but its estimate contains an arbitrary group element g and the factor $e^{-\gamma\,\mathrm{dist}(g,e)}$. The cited result and the subsequent substitution $g = g_t$ require 'for any $g \in G$'. This replacement is uniquely determined and changes no application.

Proof of Theorem 2.3: The base lattice is denoted by an undefined variable

Page 5 · thickening calculation in the proof of Theorem 2.3 · arXiv:math/0702433v4

Typo

The theorem fixes $z \in X$ and defines $\varphi(h^- h^0 hz)$, but the next three integrals contain $g_t hx$ and $g_t h^- h^0 hx$ although no x was introduced. Replacing each x by z is forced by the definition of φ and restores the displayed comparison with $I_{f,\psi}(g_t,z)$.

Equations (2.7) and the following mixing estimate: The smoothing-radius exponents have the wrong signs

Pages 5–6 · Sobolev estimates in the proof of Theorem 2.3 · arXiv:math/0702433v4

Typo

Lemma 2.2 gives a normalized bump supported at scale r the bound $\|\theta\|_\ell \ll r^{-(\ell+N/2)}$. Consequently equation (2.7) must read $\|\widetilde{\theta}\|_\ell \ll r^{-(2\ell+N/2)}$, and the mixing estimate immediately below must carry the same negative exponent. The theorem's displayed bound (2.4), the intervening line for $\|\varphi\|_\ell$, and the optimization in Remark 2.4 all already use the negative exponent, so both sign repairs are uniquely determined.

Section 4: The two error terms can be optimized simultaneously

Pages 8–10 · proof of Theorem 1.3 · arXiv:math/0702433v4

Correct and complete

The bad-set measure is a positive power of the compactness threshold, while Sobolev and injectivity losses are fixed negative powers of that threshold and the smoothing radius. Choosing both as sufficiently small exponentials in $\lfloor \mathbf{t} \rfloor$ leaves a positive exponent in every term. The finite range of smaller $\lfloor \mathbf{t} \rfloor$ is absorbed into the constant.

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. The dependence graph cites the published article, but the fixed arXiv source is the proof-bearing form previously audited in the corpus. This report reuses that exact source audit and does not claim to audit the version of record.

Sources checked

1. arXiv:math/0702433v4 · explicit fallback for version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT