MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

On Some Properties of Irrational Subspaces

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Exact version	Version of record
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Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Contains incorrect or incomplete source use

Statements

Contains unsupported statements The winning result for completely irrational matrices, its hyperplane-absolute-winning analogue, the exponent bound, and the best-approximation conclusions are correct after verified repairs to the printed arguments. The claims that the badly approximable completely irrational locus is winning and hyperplane absolute winning are not fully verified because their exact cited source chain could not be checked; no counterexample to either claim was found.

Theorems 2.1 and 2.3(1): Completely irrational matrices form winning and hyperplane-absolute-winning sets

Version of record, pp. 94–99 · Theorems 2.1 and 2.3(1), Lemmas 2.4–2.6, and proofs

Correct

For every rational complementary-dimensional subspace, nontrivial intersection with the graph of a matrix is the zero set of a nonzero determinant polynomial. The corrected algebraic-manifold arguments let Alice force a later game ball a positive distance from each such zero set. Enumerating the countably many rational subspaces and treating them successively preserves every earlier separation. This proves Theorem 2.1. The same induction in the hyperplane absolute game proves Theorem 2.3(1). The required corrections to the printed strategies are fully verified under Proofs.

Corollary 2.2 and Theorem 2.3(2): The badly approximable intersections retain an unresolved exact-source obligation

Version of record, pp. 93–94 and 99 · Propositions 1.7–1.8, Corollary 2.2, and Theorem 2.3(2)

Not able to verify

Corollary 2.2 claims that the badly approximable completely irrational locus is α-winning for every $\alpha \leq 1/2$, and Theorem 2.3(2) claims that this locus is HAW for every $n, m \in \mathbb{N}$. Both conclusions intersect the independently repaired completely irrational locus with a cited winning set of badly approximable systems. The exact 1969 Journal of Number Theory article cited for Proposition 1.7 and the exact 2013 Journal of Number Theory article cited for Proposition 1.8 could not be retrieved. The available arXiv manuscript for the latter result materially imports two lemmas from the inaccessible 1969 article, so the proof-critical chain remains unverified. This is an evidentiary limitation, not evidence that either intersection statement is false.

Theorem 3.4: The uniform-exponent bound follows after the rational-dimension index is corrected

Version of record, pp. 100–102 · Propositions 3.1–3.3, Theorem 3.4, and its example

Correct

For nonzero $\xi \in L$, the least rational subspace containing ξ has dimension $\dim_{\mathbb{Q}}(\xi)$. If this dimension were at most $d-n$, it could be extended to a rational $(d-n)$-plane meeting the completely irrational n-plane L, a contradiction. Hence $\dim_{\mathbb{Q}}(\xi) \geq d-n+1$. When $d-n \geq 2$, the ratio estimate at the actual rational dimension, its monotonicity in that dimension, and $\omega(\xi) \leq n/(d-n)$ yield the polynomial printed in the proof and the stated root bound. When $d-n=1$, the defining root is 1 and the result is the standard bound $\widehat{\omega}(\xi) \leq 1$. Thus the theorem is correct after the two adjacent index corrections recorded under Proofs.

Theorem 3.5 and Corollary 3.7: Complete irrationality forces the asserted best-approximation span

Version of record, pp. 102–103 · Theorem 3.5, Proposition 3.6, and Corollary 3.7

Correct

If all sufficiently late integer best-approximation vectors lay in a subspace of dimension at most $d - n$, their integer span could be extended to a rational $(d - n)$-plane B. Complete irrationality gives a positive angle between B and L, so $\operatorname{dist}(z_\nu, L) \geq D\|z_\nu\| \to \infty$, contradicting the defining approximation errors. Therefore $R(\Theta) \geq d - n + 1$. In the two-by-two case, the independently checked source alternative $R(\Theta) \in \{2, 4\}$ then forces $R(\Theta) = 4$.

Proofs

Contains incorrect or incomplete proofs The printed Schmidt-game and algebraic-manifold arguments contain formally invalid steps, but complete repairs establish the self-contained winning conclusions. The hyperplane-absolute proof also needs a nonzero-polynomial hypothesis and a legal deletion width. The exponent and best-approximation arguments close after a rational-dimension index repair. The two badly approximable intersection claims remain unverified through their cited inputs.

Lemma 2.4 and proof of Theorem 2.1: The printed escaping strategy is assigned to the wrong player and parameter

Version of record, pp. 91–98 · Proposition 1.4, Lemma 2.4, and proof of Theorem 2.1

Incorrect as written · verified repair

Under the paper's definition, Alice chooses the balls A_i and must force a winning outcome. Proposition 1.4 and Lemma 2.4 instead start from an Alice ball, give Bob the forcing strategy, and use $\gamma = 1 + \alpha\beta - 2\beta$, while Theorem 2.1 assumes $2\alpha < 1 + \alpha\beta$. A Bob strategy under the different condition does not prove Alice winning. Use the actual Alice escaping form: start from a Bob ball B_k, put $\gamma' = 1 + \alpha\beta - 2\alpha > 0$, and let Alice move maximally in the chosen direction until a later Bob ball lies in the required half-space. Schmidt's exact 1966 proof verifies this form. Replacing the affected A-balls by B-balls and γ by γ' throughout Lemma 2.4 preserves every radius and separation estimate and supplies precisely the strategy used in Theorem 2.1. Repair classification: Verified repair.

Algebraic-manifold induction: Zero derivatives and a false absolute-value inference require correction

Version of record, pp. 95–96 and 98–99 · proofs of Lemmas 2.4 and 2.5, especially equations (5)–(8)

Incorrect as written · verified repair

The induction demands $|\partial f/\partial z_i| > \varepsilon_{s-1}$ for every i, which is impossible when a first derivative is identically zero. A nonzero constant polynomial has empty zero set and is immediate; for a nonconstant polynomial, apply the induction only to the nonzero first derivatives. At least one exists, and after the finite sequential escapes the gradient is bounded away from zero. The published version already has the correct Taylor coefficient $1/k!$, but its remainder count d^k must be r^k, the number of ordered index choices for a polynomial in r variables. Finally, equations (7)–(8) give one-sided scalar bounds, not bounds for both absolute values. Replace the last displayed comparison by $\|y - z\| \geq (u, y - z) = (u, y - o) - (u, z - o) > \alpha\beta\gamma'\delta/8$. These corrections verify the escape estimate and apply equally to the HAW induction. Repair classification: Verified repair.

Lemma 2.5: The HAW lemma includes the zero polynomial and prints an illegal deletion width

Version of record, pp. 98–99 · Lemma 2.5 and its proof

Incorrect as written · verified repair

Lemma 2.5 quantifies over every polynomial, but for $f \equiv 0$ its zero set is all of $\mathbb{R}^r$ and no positive-distance conclusion is possible. Add the hypothesis that f is nonzero; every determinant polynomial used downstream satisfies it. The proof then chooses deletion thickness $\beta\rho_t$, although the game's rule requires it to be strictly less than $\beta\rho_t$. Choosing, for example, thickness $3\beta\rho_t/4$ is legal and, since $\rho_t > \beta\delta/2$, still exceeds the $\beta^2\delta/4$ strip containing the zero set. Together with the nonzero-derivative repair above, this completes the proof without changing Theorem 2.3. Repair classification: Verified repair.

Proof of Theorem 3.4: The rational-dimension step needs an off-by-one correction and endpoint argument

Version of record, p. 101 · first paragraph of the proof of Theorem 3.4

Incomplete as written · verified repair

The proof prints $\dim_{\mathbb{Q}}(\xi) \geq d-n$ and then invokes G_{d-n}. Complete irrationality gives $\dim_{\mathbb{Q}}(\xi) \geq d-n+1$. For $d-n \geq 2$, apply Proposition 3.3 at the actual rational dimension r and use $G_r \geq G_{d-n+1}$ for $r \geq d-n+1$; this monotonicity follows directly by substituting the root equation for G_r into the next-index polynomial. The immediately following focal polynomial already has exactly the exponents obtained from G_{d-n+1}. For $d-n=1$, Proposition 3.3 is outside its $r \geq 3$ range, but the defining root is 1 and the theorem follows from the standard bound $\widehat{\omega} \leq 1$ quoted on p. 99. Because the printed proof omits this monotonicity and endpoint treatment, the defect is substantive rather than a mere index typo. Repair classification: Verified repair.

Proof of Theorem 3.5: The angle argument covers every possible smaller eventual span

Version of record, pp. 102–103 · proof of Theorem 3.5 and deduction of Corollary 3.7

Correct and complete

The eventual span of integer best approximations is rational. If its dimension is below $d-n$, extending it to a rational $(d-n)$-plane preserves containment of the tail. Compactness of the two disjoint unit-sphere sections gives a positive angular separation, while the best-approximation errors tend to zero and their norms tend to infinity. This contradiction proves the lower bound. The exact surveyed alternative for two-by-two matrices then yields Corollary 3.7.

Corollary 2.2 and Theorem 2.3(2): The cited winning inputs could not be verified in their exact versions

Version of record, pp. 93–94 and 99 · Propositions 1.7–1.8 and their two intersection deductions

Not able to verify

The intersection arguments themselves are immediate once the two cited permanence and badly approximable winning inputs are available. The exact Schmidt 1969 version and the exact Broderick-Fishman-Simmons journal version were inaccessible, and the available manuscript for the latter still depends materially on two unverified Schmidt lemmas. Repair classification: No repair supplied for this evidentiary obligation; an exact-source check or an independent proof of those inputs is needed.

Novelty

No non-novelty findings No non-novelty findings.

Sources

Contains incorrect or incomplete source use Most proof-critical source uses are mathematically supported by exact versions or explicit fallbacks, but the Schmidt escaping input is invoked with the wrong controlling player and parameter. In addition, the exact Schmidt 1969, Broderick-Fishman-Simmons, and Kleinbock-Moshchevitin-Weiss journal versions could not all be checked. No missing proof-critical citation beyond the represented graph was identified.

Schmidt game input [8]: The escaping result is inapplicable in the form used by the focal proof

Version of record, pp. 91–98 · Propositions 1.2–1.4, Lemma 2.4, and proof of Theorem 2.1

Incorrect as invoked · verified repair

The graph terminates this branch at Schmidt's 1980 book, whose exact text was not separately audited. The focal paper states the escaping step for Bob from an Alice ball with $1 + \alpha\beta - 2\beta > 0$, then uses it as though it supplied Alice's strategy under $1 + \alpha\beta - 2\alpha > 0$. Those are different source applications. Schmidt's exact 1966 version-of-record proof independently gives the needed White/Alice form from a Bob ball with the latter parameter, so the focal theorem is repairable, but the cited input is not correctly invoked as printed.

Source: Schmidt 1966 version-of-record scan

Schmidt 1969 and Broderick-Fishman-Simmons [2,9]: The badly approximable winning inputs remain unverified in their exact cited versions

Version of record, pp. 93–94 and 99 · Propositions 1.7–1.8, Corollary 2.2, and Theorem 2.3(2)

Not able to verify

The exact 1969 and 2013 Journal of Number Theory versions were inaccessible after the recorded access checks. The exact arXiv v1 fallback for the 2013 paper does not close the obligation: its homogeneous HAW theorem uses two dimension lemmas imported from the inaccessible 1969 article. Thus the focal intersection deductions are formally applicable if those inputs hold, but the exact represented source chain could not be completed.

Source: Broderick-Fishman-Simmons arXiv v1 fallback

Broderick-Fishman-Kleinbock-Reich-Weiss [1]: The HAW permanence properties used by the focal paper are supported

Version of record, pp. 92 and 99 · Propositions 1.5–1.6 and proof of Theorem 2.3

Correct and sufficient

The exact Cambridge version of record proves countable-intersection stability for hyperplane absolute winning sets and the implication from HAW to Schmidt winning in the required parameter range. These inputs are correctly used for the focal intersection and comparison statements.

Source: Broderick-Fishman-Kleinbock-Reich-Weiss version of record

Exponent sources [4,5,7]: The imported exponent inequalities are supported, with one exact-version limitation

Version of record, pp. 100–102 · Propositions 3.1–3.3 and proof of Theorem 3.4

Supported by checked versions; exact KMW journal version unverified

The exact Marnat-Moshchevitin version of record proves the ordinary-to-uniform ratio bound. The exact Schleischitz arXiv v4 source has an incomplete proof of its full arbitrary-rational-dimension formulation, but the precise one-dimensional specialization imported here is independently obtained by rational coordinate reduction and the checked Marnat-Moshchevitin theorem. The exact Kleinbock-Moshchevitin-Weiss journal version was inaccessible; its exact arXiv v2 fallback correctly proves the ordinary-exponent bound used here. Thus the mathematical chain is supported, while equality with the inaccessible KMW version of record is not certified.

Source: Marnat-Moshchevitin version of record

Moshchevitin survey [6]: The exact best-approximation alternative used in Corollary 3.7 is supported

Version of record, p. 103 · Proposition 3.6 and Corollary 3.7

Correct and sufficient

The exact Russian Mathematical Surveys version gives Corollary 4 to Theorem 7: for a good two-by-two matrix of full rational dimension, the eventual span dimension is either 2 or 4. Complete irrationality and Theorem 3.5 exclude the first branch. The separate defect found in that survey's Theorem 68 is unrelated to this source use.

Source: Moshchevitin survey version of record

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Version of record · Uniform Distribution Theory 17(1), 89–104 (2022)
2. Schmidt 1966 version-of-record scan
3. Broderick-Fishman-Simmons arXiv v1 fallback
4. Broderick-Fishman-Kleinbock-Reich-Weiss version of record
5. Marnat-Moshchevitin version of record
6. Moshchevitin survey version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.

- Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.

- Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT