MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Effective equidistribution for closed orbits of semisimple groups on homogeneous spaces

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

Statements

Correct The effective measure-classification theorem, its topological consequence, and the stated arithmetic application are correct. The proof architecture supplies polynomial control with the advertised dependencies on the ambient groups and lattice; the source-specific notation defects recorded below are mechanical and alter no conclusion.

Theorem 1.3: Effective equidistribution toward an intermediate closed orbit

Pages 3–4 and 22–68 · Theorem 1.3 and Sections 4–16 · arXiv:0708.4040v1

Correct

Under the three hypotheses in Section 1.2, the argument alternates an effective ergodic theorem, quantitative additional invariance, and an effective closing dichotomy. Each successful enlargement strictly increases the dimension of the intermediate semisimple subgroup, so the iteration terminates after fewer than $\dim G$ steps. Taking extrema over the finite collection of possible chains makes the final Sobolev degree and exponents depend only on G and H, while the starting threshold may also depend on Γ, exactly as stated.

Theorem 1.4: The topological density conclusion follows with a polynomial radius

Pages 5 and 68–69 · Theorem 1.4 and Section 16.3 · arXiv:0708.4040v1

Correct

The proof selects a gap among the finitely many possible intermediate-orbit volumes, applies Theorem 1.3 at the upper endpoint, and tests the resulting measure estimate against a bump function of radius η around a point of the compact set F. The lower mass bound $\mu_{x_0 S}(f_x) \gg \eta^{\dim S} X^{-1}$ dominates the Sobolev error once the auxiliary integer N is sufficiently large. Compactness bounds the height uniformly, and the relation between X and V yields some $\rho > 0$ depending only on G and H.

Proposition 17.6: The prehomogeneous-hypersurface counting application

Pages 72–75 · Proposition 17.6 and Sections 17.7.1–17.7.5 · arXiv:0708.4040v1

Correct

The bounded-square-part hypothesis forces the projective height of each integral point to grow by a power of d. Proposition 17.4 converts these height bounds into lower bounds for the relevant closed-orbit volumes, including the only possible intermediate orthogonal orbit in Case B. Theorem 1.3 then gives uniform equidistribution, and the standard orbit-to-integral-point translation produces the stated power-saving count. In the $r = 8$ calculation, the printed exponent d^{10} must be d^5, as recorded in the proofs section; the ensuing weaker height bound $d^{1/8}$ and the conclusion are unchanged.

Proofs

Correct The material arguments are correct and complete after mechanically correcting the displayed matrix-coefficient moduli, two Appendix B notation slips, and one exponent in the arithmetic application. In particular, the marked spectral-gap-to-Sobolev-decay chain in Sections 6.2–6.3 and Appendix C closes with the stated parameter dependence.

Sections 4–16: The effective classification iteration preserves all dependencies

Pages 22–69 · Sections 4–16 · arXiv:0708.4040v1

Correct and complete

The Sobolev calculus in Section 5 controls point evaluation, products, translations, and smoothing. Sections 8–10 turn spectral decay into quantitative genericity and then additional almost-invariance; Sections 11–14 separate the small-periodic-orbit case from effective enlargement; and Section 15 converts almost-invariance on a closed orbit into closeness to its Haar probability. Section 16 iterates only over strictly increasing intermediate dimensions and takes uniform extrema over the resulting finite set of chains, yielding precisely Theorem 1.3.

Equations (6.1), (6.9), and (6.10): Spectral gap gives the required Sobolev matrix-coefficient decay

Pages 31–35 and 79–81 · Sections 6.2.3, 6.3.2–6.3.3, and Appendix C · arXiv:0708.4040v1

Correct and complete

For each almost-simple factor, spectral gap bounds irreducible constituents away from the trivial representation and hence gives a uniform finite L^p exponent, equivalently $1/m$-temperedness. Cowling–Haagerup–Howe majorization then yields the $\varphi_0^{1/m}$ bound for smooth vectors. For a product, every irreducible constituent has at least one tempered factor; moving the other factors to the second vector preserves the relevant Sobolev norm and gives the weak spherical-function estimate (6.10). Along a one-parameter subgroup with nontrivial projection to every simple factor, φ_0^{wk} decays exponentially, which is exactly the marked input used downstream.

Equations (6.7)–(6.10): The displayed complex matrix coefficients need modulus signs

Pages 33–34 · Equations (6.7)–(6.10) · arXiv:0708.4040v1

Typo

The four bounds print $\langle s.v, w\rangle$ on the left without absolute-value bars, although a complex matrix coefficient is not ordered. The unique correction is $|\langle s.v, w\rangle|$ in each display. The cited majorization theorem, the surrounding phrase "bounds for matrix coefficients," and every later quantitative use, including the display in Section 12.3, use the modulus. No estimate or downstream exponent changes.

Appendix B polynomial map: A stray vector symbol appears in the definition of $h(t)$

Page 78 · Appendix B, paragraph beginning "Put $h(t)$" · arXiv:0708.4040v1

Typo

The manuscript prints $h(t) = \mathrm{Ad}_{u(t)}|_V g' \in \mathrm{SL}(V)$. The symbol g' has no role and makes the definition ill-formed. Deleting it gives the unique intended polynomial map $h(t) = \mathrm{Ad}_{u(t)}|_V \in \mathrm{SL}(V)$, which is the map used in every subsequent covolume polynomial. The January 2009 author manuscript makes exactly this correction.

Appendix B intermediate Lie algebra: A tensor-product sign must be a direct-sum sign

Page 79 · Appendix B, contradiction with Lemma 3.4.1 · arXiv:0708.4040v1

Typo

After proving that the nilpotent Lie algebra $\mathfrak{l}$ is H-invariant and intersects $\mathfrak{h}$ trivially, the manuscript writes that the generated intermediate algebra is $\mathfrak{h} \otimes \mathfrak{l}$. It must be the semidirect vector-space direct sum $\mathfrak{h} \oplus \mathfrak{l}$: invariance gives $[\mathfrak{h}, \mathfrak{l}] \subseteq \mathfrak{l}$, while trivial intersection makes the sum direct. This is the nonsemisimple intermediate algebra needed for the contradiction, and the January 2009 author manuscript makes exactly this correction.

Section 17.7.5 determinant exponent: The $r = 8$ covariant determinant has exponent 5, not 10

Page 75 · Section 17.7.5, first paragraph of the proof in Case B · arXiv:0708.4040v1

Typo

The paragraph prints that $\det(\mathrm{cov}(y))$ is a fixed multiple of d^3 or d^{10} for $r = 7$ or 8. Section 17.7.4 immediately establishes the respective identities $\det(\mathrm{cov}(t)) \asymp \mathrm{disc}(t)^3$ and $\det(\mathrm{cov}(t)) \asymp \mathrm{disc}(t)^5$, so the unique correction is d^5. The following lower bound $d^{1/8}$ is weaker than the corrected determinant estimate supplies, hence no later claim changes.

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. Springer's version of record is subscription-controlled. arXiv exposes only the fixed v1 manuscript from August 30, 2007. A January 10, 2009 author-hosted manuscript is materially revised but is not identified as the version of record; it was used only to confirm version changes and mechanically intended corrections. This report therefore evaluates arXiv v1 and does not claim to audit the VOR.

Sources checked

1. arXiv:0708.4040v1 · explicit fallback for inaccessible version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT