MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Badly approximable systems of affine forms and incompressibility on fractals

Ryan Broderick, Lior Fishman, David Simmons

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Exact version	arXiv:1208.2091v1 (explicit fallback)
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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

Statements

Contains unsupported statements The lacunary escaping-set theorem, the fixed-matrix theorem, its incompressibility corollary, and the negative diffuse-set example are correct after verified repairs to three internal proof defects. The hyperplane-absolute-winning theorem for homogeneous systems, and the fixed-shift corollary that builds on it, are not able to be fully verified from the available exact sources because their proof chain materially imports Schmidt's inaccessible 1969 lemmas; no counterexample to either statement was found.

Theorems 4.1 and 1.1: Lacunary escaping sets and fixed-matrix affine slices are HAW

Pages 2 and 7–8 · Theorems 1.1 and 4.1 and proof · arXiv:1208.2091v1

Correct

For each norm window, lacunarity leaves at most n dangerous indices. Each corresponding inverse image of a target ball lies in a hyperplane neighborhood of thickness at most $\beta^r \rho(B_j)$ after replacing the printed affine-plane definition by the verified scalar-projection definition recorded in the proofs section. Repeating the percentage deletion for $r = \lfloor \log_2 n \rfloor + 1$ moves eliminates all dangerous neighborhoods and gives a uniform positive avoidance constant. Lemma 2.1 then transfers the strategy to the hyperplane absolute game. For an irrational fixed matrix, the exact cited Cambridge article supplies a lacunary sequence Y with $\widetilde{E}(Y, \mathbb{Z}) \subseteq \mathrm{Bad}_A(M, N)$. In the complementary rational case, that source shows that $\mathrm{Bad}_A(M, N)$ contains the complement of a countable family of parallel affine hyperplanes, which is HAW by the elementary one-hyperplane strategy and countable-intersection stability. Upward closure of HAW proves Theorem 1.1 in both cases.

Source: Exact arXiv v1 manuscript

Corollary 1.2 and Example 3.2: The incompressibility consequence and the diffuse-set obstruction are correct

Pages 2 and 6 · Corollary 1.2, Proposition 3.1, and Example 3.2 · arXiv:1208.2091v1

Correct

The cited HAW permanence results give countable-intersection stability, nonsingular C^1 invariance, winning on hyperplane-diffuse sets, and full dimension on Ahlfors-regular supports, which yield Corollary 1.2 with its stated quantifiers. In Example 3.2, the three similarities satisfy the open set condition and have noncollinear fixed points. The recursive separation gives an absolute slope bound of 5 on pairs of points in the limit set, so the set lies on a globally extended 5-Lipschitz graph; the shear $\Phi(x, y) = (x, y + f(x))$ is bi-Lipschitz and maps the horizontal axis over that graph. This proves the claimed obstruction.

Source: Cambridge HAW permanence source

Theorem 1.3: The homogeneous-system HAW conclusion has an unresolved source obligation

Pages 2 and 9–14 · Theorem 1.3, Propositions 5.1–5.2, and Sections 5–7 · arXiv:1208.2091v1

Not able to verify

The determinant-avoidance construction, after the verified repairs recorded in the proofs section, reduces the theorem to Propositions 5.1 and 5.2. Those propositions are explicitly Schmidt's Lemmas 1 and 2 from the 1969 Journal of Number Theory article and supply the dimension bounds that place each family of dangerous integer solutions in the required finite-dimensional subspace. The exact cited version of that article could not be obtained, so this proof-critical input could not be checked. This is an evidentiary limitation, not evidence that either imported lemma or Theorem 1.3 is false.

Source: Publisher record for Schmidt's 1969 article

Corollary 1.4: The fixed-shift HAW assertion is not established in the reviewed version

Page 2 · Corollary 1.4 and the sentence immediately following it · arXiv:1208.2091v1

Not able to verify

The manuscript states that the fixed-shift result follows by combining additional steps from Einsiedler–Tseng with the proof of Theorem 1.3, but it omits that adaptation. The cited paper proves a Schmidt-winning result, whereas upgrading the modified construction to a hyperplane-absolute strategy requires substantive compatibility checks that are not supplied here. The route also inherits the unresolved Schmidt-lemma obligation in Theorem 1.3. No counterexample was found, but the available material does not provide a complete verification.

Source: Einsiedler–Tseng cited source

Proofs

Contains incorrect or incomplete proofs Three proof arguments are incorrect as written: Theorem 4.1 defines an affine preimage that can be empty, Lemma 7.2 equates a largest minor with the Euclidean norm of all minors, and the induction proving Lemma 6.1 uses a nonexhaustive case split and false norm/diameter constants. All three have complete verified repairs that preserve the advertised statements. The proof of Theorem 1.3 still has an unverified exact-source dependency, and Corollary 1.4 omits a substantive adaptation. Four mechanical notation corrections are harmless.

Theorem 4.1, Equations (4.17)–(4.19): The printed inverse-image affine hyperplane can be empty

Page 8 · proof of Theorem 4.1, paragraph between Equations (4.18) and (4.19) · arXiv:1208.2091v1

Incorrect as written · verified repair

The target point $y_k \in Z_k$ need not lie in $\mathrm{im}(M_k)$, so $M_k^{-1}(y_k)$ can be empty and the printed set $L = M_k^{-1}(y_k) + W$ is then not an affine hyperplane. Define instead

$$L = \left\{x \in \mathbb{R}^N : \langle M_k x - y_k, M_k v_k\rangle = 0\right\}.$$

Its normal is $M_k^\top M_k v_k$, and $\|M_k^\top M_k v_k\| \geq \langle M_k^\top M_k v_k, v_k\rangle = t_k^2$. Consequently, if $\|M_k x - y_k\| < c$, then

$$\mathrm{dist}(x, L) \leq \frac{c\, t_k}{\|M_k^\top M_k v_k\|} \leq \frac{c}{t_k}.$$

This proves exactly the containment needed in (4.19) for every target point, including points outside the image, and the remainder of the percentage-game argument is unchanged. Repair classification: Verified repair.

Source: Exact arXiv v1 manuscript

Lemma 7.2: A largest minor is incorrectly identified with the Euclidean norm of all minors

Pages 12–13 · Lemma 7.2 and its proof · arXiv:1208.2091v1

Incorrect as written · verified repair

After choosing ω' to maximize $|D_{\omega'}(A)|$, the proof uses $|D_{\omega'}(A)| = M_{v-1}(A)$. With the manuscript's Euclidean norm this equality is generally false. Since $|\Omega_{v-1}| = \binom{N}{v-1}^2$, the valid bound is

$$|D_{\omega'}(A)| \geq \frac{M_{v-1}(A)}{\binom{N}{v-1}}.$$

Put $d_v = \binom{N}{v-1}$. The displayed directional-derivative calculation then gives at least $(d_v^{-1} - \sqrt{N}\,\varepsilon_1)M_{v-1}(A)$. Choosing $\varepsilon_1 \leq (2\sqrt{N}\,d_v)^{-1}$ and $\varepsilon_2 \leq [2d_v(1+\sqrt{N}\,\sigma)]^{-1}$ proves the lemma with the required uniform positive constants. Every later use needs only their positivity. Repair classification: Verified repair.

Source: Exact arXiv v1 manuscript

Lemma 6.1 induction: The case split and ball-diameter estimates are invalid as printed

Pages 13–14 · Claim 7.3 and Cases 1–2 in the proof of Lemma 6.1 · arXiv:1208.2091v1

Incorrect as written · verified repair

The two cases are stated on the original ball B, but Case 2 later needs its witness A to lie in B_{v-1} in order to bound $\|A' - A\|$; thus the printed cases do not support the subsequent Taylor estimate. Moreover, two points of a ball of radius below $\mu_{v-1}\rho_B$ can be $2\mu_{v-1}\rho_B$ apart, not $\mu_{v-1}\rho_B$ apart as used in Claim 7.3 and the quadratic remainder. A complete repair is to split the cases on B_{v-1} and put $d_v = \binom{N}{v-1}$ and $C_v = 1 + 2d_v(v-1)$. Applying the mean-value estimate componentwise to the d_v^2 minors and then the Euclidean triangle inequality gives $M_{v-1}(B_{v-1}) \leq C_v M_{v-1}(A)$. In Case 2, choose

$$\mu_{v-1} = \frac{\beta^2 \varepsilon_2 \nu_{v-1}}{4v^2};$$

the first-crossing property gives $\rho(B_{v-1}) \geq \beta\mu_{v-1}\rho_B$, so the linear term is at least $\beta^2\varepsilon_2\mu_{v-1}\rho_B M_{v-1}(A)$, while the corrected quadratic remainder is at most half of it. Taking ν_v no larger than both ε_1/C_v and $\beta^2\varepsilon_2\mu_{v-1}/(2C_v)$ proves (6.33) in both cases; the unnecessary printed σ denominator in Case 1 is simply removed because $\rho_B < 1$. Repair classification: Verified repair.

Source: Exact arXiv v1 manuscript

Propositions 5.1–5.2: The imported Schmidt dimension bounds could not be checked in the exact cited source

Page 9 · Propositions 5.1–5.2; used on pages 10–12 in Lemmas 5.3–5.4 and Theorem 1.3 · arXiv:1208.2091v1

Not able to verify

These propositions are the only input that bounds the spans of the dangerous integer vectors by N and M, respectively. The later determinant strategy uses those bounds decisively to choose the orthonormal systems on which Lemma 6.1 operates. The manuscript cites Schmidt's Lemmas 1 and 2 in the 1969 Journal of Number Theory paper, but the exact article could not be retrieved and no legitimate exact scan was available. Repair classification: No repair supplied; the source statements must be checked in that cited version or independently reproved. This does not show that they are false.

Source: Publisher record for the inaccessible cited article

Corollary 1.4: The HAW adaptation for a fixed inhomogeneous shift is omitted

Page 2 · Corollary 1.4 and following sentence · arXiv:1208.2091v1

Incomplete as written

The proof consists only of the assertion that additional steps from Einsiedler–Tseng can be combined with the proof of Theorem 1.3. The cited winning strategy is not itself a hyperplane-absolute strategy, and the manuscript does not identify the modified dangerous families or verify that each is captured by the single hyperplane deletions required here. This is a substantive construction, not an immediate formal consequence of the two cited arguments. Repair classification: No repair supplied.

Source: Einsiedler–Tseng cited source

Lemma 2.1 and Theorem 4.1 strategy: The percentage-to-absolute conversion and window deletion are otherwise complete

Pages 4–5 and 7–8 · Lemma 2.1 and proof of Theorem 4.1 · arXiv:1208.2091v1

Correct and complete

Repeating one percentage move for m turns leaves at most a proportion $(1-p')^m \leq 1-p$ of its neighborhoods, and the finite-cover argument converts a percentage move into one legal absolute deletion. In Theorem 4.1, a norm window contains at most n indices, while $r = \lfloor \log_2 n \rfloor + 1$ successive half-deletions leave fewer than one dangerous hyperplane. After the affine-plane repair above and the radius correction below, all neighborhood widths are legal and the uniform constant c is independent of the window.

Source: Exact arXiv v1 manuscript

Four local notation corrections: A radius factor, two indices, and one sign need mechanical correction

Page 7 · Equation (4.15) and paragraph after (4.14); page 8 · paragraph before (4.19); page 13 · proof of Lemma 7.2 · arXiv:1208.2091v1

Typo

First, (4.15) must read $\rho_1 < \beta^r\delta/(4t_1)$, not $\rho_1 < \beta^r\delta t_1/4$; the reciprocal is forced by the next estimate $\delta\beta^{rj}/(2t_1) \geq 2\rho(B_j)$. Second, the sentence after (4.14) should say that t_k lies in the jth window, not the kth window. Third, in the decomposition $x = w + \eta v_k$ the conclusion is $|\eta| > c/t_k$, not $\eta > c/t_k$. Fourth, the sum defining $A'e_{i_0}$ in Lemma 7.2 is indexed by i, so its displayed lower limit must be $\sum_{i=1}^{N}$, not $\sum_{j=1}^{N}$. Each correction is uniquely determined by the surrounding calculation and changes no conclusion.

Source: Exact arXiv v1 manuscript

Novelty

No non-novelty findings No non-novelty findings.

Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. The Journal of Number Theory version of record was access-controlled and could not be retrieved for this audit; the report therefore evaluates only the exact arXiv v1 manuscript and does not claim to audit the VOR.

Sources checked

1. Exact arXiv v1 manuscript
2. Cambridge HAW permanence source
3. Publisher record for the inaccessible cited article
4. Einsiedler–Tseng cited source

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT