MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

The set of badly approximable vectors is strongly C^1 incompressible

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

Statements

Correct The hyperplane-absolute-winning result for badly approximable vectors, its C^1 invariance, the strong incompressibility consequence, the fractal-intersection results, and the nondense-orbit example are correct. Three uniquely determined local typographical corrections do not alter any statement.

Theorems 2.4 and 2.5: Hyperplane absolute winning is C^1 invariant and contains the badly approximable vectors

Journal pages 323–327 · Theorems 2.4–2.5 and their proofs · version of record

Correct

For Theorem 2.4, uniform control of Df and $D(f^{-1})$ on the first sufficiently small compact game ball permits a subsequence of Bob's moves to be mapped to a legal game with parameter $\beta' = \beta^n$. The affine linearization error is made smaller than the deletion margin, so pulling a deleted k-plane neighborhood back gives a legal deletion and transfers Alice's winning strategy. For Theorem 2.5, the simplex lemma confines every denominator block to one affine hyperplane, and deleting its neighborhood yields the uniform bound $\|x - p/q\| \geq cq^{-(1+1/d)}$. The printed move index A_{j_k+1} must be A_{j_k}; with this mechanical correction the deletion is made immediately after B_{j_k}, has admissible width $\beta^{k+1}\rho < \beta\rho_{j_k}$, and forces B_{j_k+1} to avoid the dangerous rational points.

Source: Cambridge version of record

Theorems 1.1 and 1.2: The strong incompressibility and diffuse-fractal conclusions follow

Journal pages 320–321 and 330–334 · Theorems 1.1, 1.2, 4.10 and Corollary 5.4 · version of record

Correct

Hyperplane absolute winning is preserved by countable intersections and by inverse images under nonsingular C^1 maps. Proposition 4.9 transfers the game to every hyperplane-diffuse closed set, and the winning-on-K dimension lemma gives positive dimension. When K supports an Ahlfors-regular absolutely decaying measure, its Hausdorff dimension is recovered in full. Applying these facts to BA_d proves both advertised incompressibility conclusions with the stated quantifiers over the maps and open set.

Source: Cambridge version of record

Theorem 2.6: Avoidance sets for semisimple toral endomorphisms are HAW

Journal pages 324 and 327–330 · Theorem 2.6 and proof · version of record

Correct

If the spectral radius is one, Kronecker's theorem and semisimplicity make a common power of the integral rational matrix equal to the identity, reducing the target to avoidance of a countable set. If the spectral radius exceeds one, the proof separates the maximal-modulus invariant subspace, compares different inverse images of the target lattice, and deletes the affine hyperplane parallel to the complementary invariant subspace at each selected scale. The separation estimates ensure that every surviving orbit stays a fixed distance from the target point.

Source: Cambridge version of record

Proofs

Correct The material proofs are correct and complete after three mechanical typographical corrections. The pullback strategy, simplex-lemma argument, diffuse-set restriction, and dimension deductions preserve all parameter ranges and quantifiers.

Proof of Theorem 2.4: The game pullback controls nonlinear distortion at every selected scale

Journal pages 324–327 · proof of Theorem 2.4 · version of record

Correct and complete

The choice $C = 2C_1C_2$ and n with $C(1+\beta)\beta^{n-2} < 1$ makes the pulled-back deletion thinner than $\beta\rho_i$. The chosen subsequence has radius ratio in $[\beta^n, \beta^{n-1})$, which supplies a legal β^n-game and leaves the required separation from the preceding deleted plane. Continuity of the inverse derivative makes the affine approximation error uniform on all later balls. Thus every outcome of the original game maps into an outcome of the transferred winning strategy.

Source: Cambridge version of record

Proof of Theorem 2.5: The deleted-plane move has an off-by-one printed index

Journal page 327 · proof of Theorem 2.5, display defining Alice's move · version of record

Typo

The display prints $A_{j_k+1} = \mathcal{L}_k^{(\beta^{k+1}\rho)}$, although Alice must answer Bob's already chosen ball B_{j_k}. The unique correction is $A_{j_k} = \mathcal{L}_k^{(\beta^{k+1}\rho)}$. It is legal because $\rho_{j_k} > \beta^k\rho$, and then the next ball B_{j_k+1} avoids the neighborhood exactly as the following sentence requires. No theorem or later application changes.

Source: Cambridge version of record

Proposition 5.1: The displayed strict comparison should be an equality

Journal page 334 · proof of Proposition 5.1 · version of record

Typo

After choosing $\beta = (1/C)^{1/\gamma}$, the proof says $C\beta^\gamma < 1$; the left side is exactly 1. Replacing $<$ by $=$ is the unique correction. The defining decay estimate itself is strict, so it still gives $\mu(B(x,\rho) \cap \mathcal{L}^{(\beta\rho)}) < \mu(B(x,\rho))$, leaving a support point outside the hyperplane neighborhood and proving diffuseness without any additional argument.

Source: Cambridge version of record

Lemma 5.3: The open-set variable is misnamed once

Journal page 334 · statement of Lemma 5.3 · version of record

Typo

The lemma quantifies an open set U but ends its hypothesis with $V \cap K \neq \varnothing$. The unique intended correction is $U \cap K \neq \varnothing$, as in the preceding and following statements. The corrected hypothesis is exactly the one used to deduce Corollary 5.4.

Source: Cambridge version of record

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. Cambridge version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT