MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Effective multiple equidistribution of translated measures

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Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

Statements

Correct The abstract multiple-equidistribution theorem and its homogeneous-space specializations are correct. Theorem 1.1 is specifically the $\mathrm{SL}_{m+n}(\mathbb{R})/\mathrm{SL}_{m+n}(\mathbb{Z})$ compact-orbit case; Theorem 1.3, not Theorem 1.1, is the paper's general semisimple/parabolic result.

Theorem 1.1: The first main theorem is the special matrix-lattice case

Journal page 213 · Theorem 1.1 · version of record

Correct

The theorem concerns a Wiener probability measure on a compact orbit of $U_{m,n}$ in $\mathrm{SL}_{m+n}(\mathbb{R})/\mathrm{SL}_{m+n}(\mathbb{Z})$. It bounds the r-fold translated correlation by Sobolev norms times exponential decay in the minimum of the distances to the expanding-cone boundary and the pairwise parameter distances. Those are exactly the hypotheses and conclusion obtained from the later abstract theorem in this special setting.

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Theorem 1.3: The general semisimple/parabolic result is Theorem 1.3

Journal pages 213–215 · Theorem 1.3 and deduction from Theorem 2.1 · version of record

Correct

Here G is connected semisimple without compact factors, P is a parabolic subgroup whose projection to each simple factor is proper, U is its abelian unipotent radical, and the Wiener measure is supported on a compact U-orbit. This is the theorem that supplies effective multiple equidistribution in the general expanding-cone setting. The version of record explicitly distinguishes it from Theorem 1.1; a citation to Theorem 1.1 for this general scope is therefore a locator error in the citing paper, not a defect in this source.

Source: IMRN version of record

Theorem 2.1: The abstract correlation estimate follows from the stated equidistribution inputs

Journal pages 214–215 · assumptions (EQ1)–(EQ2) and Theorem 2.1 · version of record

Correct

For a U-ergodic measure with discrete spectrum, the theorem assumes effective one-translate equidistribution, polynomial mixing along U, and explicit algebra and norm controls. It concludes decay for every higher translated correlation in the minimum of the individual equidistribution scales and pairwise T-separations. The proof's induction records every Sobolev-degree and exponent loss, so the constants depend only on the announced order and norm data.

Source: IMRN version of record

Proofs

Correct The Fourier reduction, one-factor estimate, root-direction selection, and recursive higher-correlation bound are correct and complete. Two uniquely determined variable-name errors are harmless.

Sections 3–4: Fourier expansion and the base estimate correctly start the induction

Journal pages 215–223 · Sections 3–4.3 · version of record

Correct and complete

The discrete U-spectrum expands the Wiener density into eigencharacters with absolutely summable coefficients. For a nontrivial eigencharacter, averaging along a one-parameter subgroup converts its oscillation into a term controlled jointly by (EQ1) and (EQ2); optimizing the averaging length gives a positive decay exponent. A root direction maximizing the relative adjoint expansion of the translate parameters is then selected, and its ordered expansion sizes provide the scale separation required for the inductive step.

Source: IMRN version of record

Proposition 4.4 and Lemma 4.5: The recursive error decomposition closes for every correlation order

Journal pages 225–232 · Proposition 4.4, Lemma 4.5, and proof of Theorem 2.1 · version of record

Correct and complete

Proposition 4.4 decomposes the averaged correlation at a split index into an averaging error, a mixing error, and products of lower-order errors, with the announced Sobolev losses. Lemma 4.5 uses the ordered adjoint-expansion sizes to choose a split whose adjacent gap is large relative to the overall separation. Choosing the averaging length as a power of that gap makes every term a negative power of the controlling scale; induction on r then proves Theorem 2.1.

Source: IMRN version of record

Lemma 4.1 proof: An undefined test-function symbol should be φ

Journal page 220 · proof of Lemma 4.1 · version of record

Typo

The third expression in the displayed computation ends with the undefined factor ψ, in $\nu(\psi_\xi \circ \exp(-w) \cdot \psi)$. The unique correction is to replace that last factor by the test function φ. Then U-invariance and the eigenfunction identity give $\exp(w)_* \nu_\xi = \xi(-w)\nu_\xi$ exactly as the following line states.

Source: IMRN version of record

Introductory description of the decay parameter: The final translate parameter is misindexed

Journal page 212 · paragraph immediately before Theorem 1.1 · version of record

Typo

The paragraph says that the improved estimate depends on $t_1, \ldots, t_1$ and the pairwise distances. The unique correction is $t_1, \ldots, t_r$, as shown in the definition of the decay parameter immediately below and in every theorem and proof use. No mathematical assertion changes.

Source: IMRN version of record

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. IMRN version of record

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT