MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Central limit theorems for group actions which are exponentially mixing of all orders

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Reviewed source	arXiv:1706.09167v1 · explicit fallback for the inaccessible Journal d'Analyse Mathématique version of record
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Audit date	August 23, 2026
Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

Statements

Correct The central-limit theorem for Følner averages of actions that are exponentially mixing of all orders, the general measure-averaging criterion, and the homogeneous and lacunary-unipotent applications are correct. One variance display has a uniquely determined normalization typo.

Theorem 1.1 and Corollary 1.3: Subexponential-growth averages satisfy the asserted central-limit theorem

PDF pages 3–4 and Sections 4–5 · Theorem 1.1, Corollary 1.3, and proofs · arXiv:1706.09167v1

Correct

For a right Følner sequence in a group of subexponential metric-volume growth, the normalized Haar measures satisfy the local-mass condition of Theorem 1.5. Exponential two-point mixing makes the covariance integrable, and the Følner property identifies the limiting variance as $\int_H \langle f, f \circ h^{-1} \rangle \, dm_H(h)$. The cumulant theorem then gives convergence to the centered Gaussian with this variance. The Cartan-action corollary uses the stated strong-spectral-gap input to supply exponential mixing of all orders.

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Theorem 1.5: The general central-limit criterion is correct after a variance-notation correction

PDF page 4 · equation (1.7) in Theorem 1.5 · arXiv:1706.09167v1

Typo

The theorem prints $\sigma_f := \lim_T \|\nu_T * f\|^2_{L^2}$ and then denotes the limiting Gaussian by $N(0, \sigma_f^2)$. The unique correction, confirmed by the proof and Proposition 5.1, is $\sigma_f^2 := \lim_T \|\nu_T * f\|^2_{L^2}$. With that correction, the local-mass hypothesis kills every cumulant of order at least three and the assumed second-moment limit is exactly the Gaussian variance.

Source: Exact arXiv version 1

Corollary 1.6: Lacunary sampling of a unipotent flow has the stated Gaussian limit

PDF pages 4–5 and Section 5 · Corollary 1.6 and proof · arXiv:1706.09167v1

Correct

For the normalized counting measures on lacunary times, any logarithmic metric ball contains only a uniformly bounded number of sample points relative to the required normalization. Polynomial multiple mixing of unipotent translates becomes exponential in the sample index because the times are lacunary. Distinct off-diagonal correlations vanish in the variance computation, leaving $\|f\|^2_{L^2}$, as claimed.

Source: Exact arXiv version 1

Proofs

Correct The cumulant decomposition, separation estimates, local-mass argument, and application proofs are correct and complete. Proposition 6.2—the combinatorial decomposition used by the focal paper—is proved directly. One omitted total-mass factor in an intermediate displayed bound is a mechanical typo; the proposition statement and its application contain the factor.

Theorem 1.5 proof: Clustered and separated tuples give vanishing higher cumulants

PDF pages 9–15 · Sections 5–9 and proof of Theorem 1.5 · arXiv:1706.09167v1

Correct and complete

The joint cumulant is integrated over H^r. Tuples inside a logarithmic cluster are controlled by the assumed local mass, while tuples split into well-separated blocks have exponentially small conditional cumulants by multiple mixing and the seminorm growth estimates. The recursively chosen separation scales make both contributions tend to zero. Proposition 5.1 then converts vanishing cumulants and convergence of the second moment into the claimed Gaussian limit.

Source: Exact arXiv version 1

Proposition 6.2: The finite cluster decomposition of H^r is valid

PDF page 12 and PDF pages 18–20 · Proposition 6.2 and Section 10 proof · arXiv:1706.09167v1

Correct and complete

Starting from the discrete partition of $[r]$, if two blocks are not separated at the next threshold the proof merges them. The triangle inequality gives diameter below three times that threshold for the merged block. Each failure therefore produces a strictly coarser partition, so after at most $r-1$ mergers either the whole tuple is clustered or it belongs to one of the announced separated-block regions. This proves the exact covering used in the cumulant estimate and in the focal argument.

Source: Exact arXiv version 1

Proof of Proposition 5.2: The last intermediate estimate omits the measure's total-mass factor

PDF page 12 · final display in the proof of Proposition 5.2 · arXiv:1706.09167v1

Typo

After bounding the conditional cumulant uniformly by $e^{-\delta_r \gamma} N_s(f)^r$, the integral over a separated region is printed with the same bound but without its measure. The unique correction is to multiply that term by $\|\nu_T\|^r$, since the region has $\nu_T^{\otimes r}$-measure at most $\|\nu_T\|^r$. Proposition 5.2 itself includes this factor, and the next proof substitutes the corrected stated estimate, so the argument is unchanged.

Source: Exact arXiv version 1

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. The publisher version of record was not publicly downloadable during this audit, and no independently authenticated author manuscript matching the journal pagination was available. The complete arXiv version 1 was reviewed as the explicit open proof-bearing fallback; this report does not claim inspection of the publisher file.

Sources checked

1. arXiv:1706.09167v1 · explicit fallback for the inaccessible Journal d'Analyse Mathématique version of record
2. Exact arXiv version 1

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT