MATHAUDIT / PUBLISHED-PAPER REPORT

Correctness, proof completeness, and novelty audit

Quantitative multiple mixing

Michael Björklund, Manfred Einsiedler, Alexander Gorodnik

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Audit date	August 23, 2026
Scope	Statements, proofs, and novelty

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

Statements

Correct The exponential multiple-mixing theorems for real, S-algebraic, and adelic homogeneous actions, the coupling-uniform forms, and the approximate-configuration consequence are correct at their printed scope. In particular, the real homogeneous-space theorem explicitly assumes strong spectral gap; the paper never asserts that result without this hypothesis.

Theorems 1.1 and 1.2: Real multiple mixing is stated with the necessary strong-spectral-gap hypothesis

PDF pages 2–4 · definition of strong spectral gap and Theorems 1.1–1.2 · arXiv:1701.00945v2

Correct

The action of the connected semisimple group G on $X = \Gamma\backslash L$ is expressly required to have strong spectral gap, meaning that the representation on $L^2_0(X)$ restricts to every noncompact simple factor with the trivial representation isolated. Under precisely that hypothesis, Theorem 1.1 bounds every k-point correlation by an exponentially decaying function of the minimum pairwise group distance, and Theorem 1.2 gives the same estimate uniformly over diagonal-invariant couplings. The binary matrix-coefficient estimate used to start the proof is available under that same hypothesis, so the statement and its proof have matching scope.

Source: Final arXiv version 2

Corollary 1.3: The approximate lattice-configuration consequence follows

PDF pages 4–5 and Section 3 · Corollary 1.3 and proof · arXiv:1701.00945v2

Correct

For an irreducible lattice in a semisimple group without compact factors, the required strong spectral gap is available. Smooth approximate identities on the quotient have controlled Sobolev norms; applying Theorem 1.1 to their translates makes the multiple correlation positive once the minimum pairwise distance is a sufficiently large multiple of $\log(1/\varepsilon)$. Unfolding that positive integral produces the asserted lattice elements and common translate with error below ε.

Source: Final arXiv version 2

Theorems 1.4 and 1.5: The arithmetic multiple-mixing estimates have the stated abstract hypotheses

PDF pages 5–7 and Section 4 · Theorems 1.4–1.5 and proofs · arXiv:1701.00945v2

Correct

The S-algebraic statements assume explicit Sobolev properties and a quantitative two-point mixing bound. The proof uses only those inputs to obtain decay in the minimum pairwise height, first for ordinary correlations and then uniformly for diagonal-invariant couplings. The compact-open invariance and Sobolev-degree losses are retained in all quantified constants.

Source: Final arXiv version 2

Theorem 1.6: The adelic multiple-mixing conclusion is supported

PDF pages 7–8 and Section 5 · Theorem 1.6 and proof · arXiv:1701.00945v2

Correct

For a simply connected absolutely simple group that is isotropic at an Archimedean place, the proof divides according to whether the Archimedean separation is large. The large case applies the uniform S-algebraic coupling estimate, while the complementary case invokes effective equidistribution of the relevant closed orbit; balancing the two errors gives the claimed decay in adelic height. The typographical word correction recorded in the proof section does not change this hypothesis.

Source: Final arXiv version 2

Proofs

Correct The coupling induction, its real and arithmetic specializations, and the adelic case split are correct and complete. Two uniquely determined notation errors are harmless and do not lower the overall proof status.

Sections 2 and 6: The coupling induction closes with compatible scale choices

PDF pages 9–20 and 36–42 · reduction to and proof of the general coupling estimate · arXiv:1701.00945v2

Correct and complete

The argument partitions the indices into two clusters, averages along a root direction that separates the clusters, and decomposes the correlation error into slow-motion, two-point-mixing, and lower-order-coupling terms. Sobolev growth is polynomial in the averaging scale, whereas the available mixing term decays exponentially; the selected scale balances these terms. Induction on the number of factors then yields decay in the minimum pairwise separation without losing uniformity in the coupling.

Source: Final arXiv version 2

Sections 4 and 5: The arithmetic and adelic reductions preserve all required uniformity

PDF pages 23–35 · arithmetic and adelic proofs · arXiv:1701.00945v2

Correct and complete

The S-algebraic proof verifies that height and adjoint growth provide the same three estimates used in the real coupling argument. In the adelic proof, the large-Archimedean branch uses that uniform estimate on a projected coupling and the small-Archimedean branch applies the cited effective closed-orbit equidistribution theorem with its isotropy hypothesis. The final choice of the splitting parameter makes both branches decay by a positive power of the minimum pairwise height.

Source: Final arXiv version 2

Equations (1.2) and (1.3): Two matrix-coefficient inequalities omit modulus signs

PDF page 2 · equations (1.2)–(1.3) · arXiv:1701.00945v2

Typo

The left sides are printed as $\langle \pi(g)v_1, v_2\rangle$ in inequalities whose right sides are nonnegative real quantities. The unique correction is $|\langle \pi(g)v_1, v_2\rangle|$ in both displays. The subsequent correlation estimate is written with modulus, and every application uses the absolute-value form, so no argument or theorem changes.

Source: Final arXiv version 2

Theorem 1.6 hypothesis: The word “isotopic” should be “isotropic”

PDF page 7 · statement of Theorem 1.6 and following discussion · arXiv:1701.00945v2

Typo

The hypothesis twice says that the algebraic group is “isotopic” over an Archimedean completion. The unique mathematical term required by the cited effective-equidistribution input is “isotropic.” The surrounding discussion and the proof identify that same standard isotropy hypothesis, so the correction is mechanical and harmless.

Source: Final arXiv version 2

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

The cited journal version of record was not accessible to the auditor. This audit therefore evaluated only the explicitly linked arXiv version. It does not claim to audit the inaccessible version of record. The exact JEMS version of record was not publicly downloadable during this audit. The complete final arXiv version 2 was therefore reviewed as the explicit open proof-bearing fallback; this report does not claim inspection of the publisher file.

Sources checked

1. arXiv:1701.00945v2 · explicit fallback for the inaccessible JEMS version of record
2. Final arXiv version 2

APPENDIX

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT