

Correctness, proof completeness, and novelty audit

Deformations of the braid arrangement and trees

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Scope	Statements, proofs, and novelty
Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny.	

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

Statements

Correct The boxed-tree formulas for region counts and coboundary polynomials, the transitive-arrangement equinumeracy results, and the region-to-tree bijection of Theorem 8.8 are correct. The published proof of the last result contains a defective explicit inverse construction, but the repaired construction in arXiv:1604.06554v4 verifies the theorem rather than changing its statement.

Theorems 4.2, 4.6, and 5.2: The boxed-tree counting and coboundary formulas are supported

Journal pages 478–502 · Theorems 4.2, 4.6, 5.2 and Sections 5–7

Correct

Central subarrangements are encoded by weighted graphs, the multivariate exponential formula isolates connected components, and the gas/tree decomposition evaluates the resulting formal series. Zaslavsky’s characteristic-polynomial evaluation then converts the coboundary identity into the signed region count. For transitive tuples, the local cadet condition makes the remaining signed terms cancel in pairs, leaving exactly the claimed family of trees. The formal-series limits are coefficientwise and the finite-support estimates used before taking them are adequate.

Source: [Exact Advances in Mathematics version of record](#)

Proposition 8.1: Annotated sketches are bijective with labeled plane trees

Journal pages 506–507 · Proposition 8.1 and proof

Correct

The bud-growth construction creates non-bud vertices in the tree order used by the proposed inverse, so reading the children in that order recovers every annotated sketch. Conversely, annotation reconstructs the same growth sequence. The consecutive-letter clause follows directly from whether the relevant child is a node or a leaf. The proof contains harmless local symbol slips—its input word belongs to $D^{(m)}(n)$, and the final child test uses the exponent of the following letter—but these have unique corrections and do not affect the proposition.

Theorem 8.8: The general transitive region bijection is correct after the Section 8.1 repair

Journal pages 503–513 · property (iv), Proposition 8.1, Lemmas 8.12–8.13, and Theorem 8.8

Correct after verified repair

Catalan regions are encoded by annotated sketches and then by trees. Lemma 8.12 gives a unique S -maximal Catalan subregion in every S -region, while Lemma 8.13 identifies S -local maximality with membership in T_S . Thus Ψ_S is surjective for every S . When S is transitive, Theorem 4.6 gives equal finite cardinalities and promotes the surjection to a bijection. The only material defect is the explicit inverse recipe in property (iv); replacing its fixed increments by the decreasing increments printed in arXiv v4 makes the Catalan-region encoding bijective and completes the chain.

Source: [Corrected arXiv version 4](#)

Proofs

Contains incorrect or incomplete proofs The version of record’s explicit inverse in Section 8.1 is not a valid construction:

its fixed increments can make two successive ordered values equal. This affects the published proof of the Catalan-region/sketch bijection and hence the setup of Theorem 8.8. The defect has a verified repair in arXiv version 4; the rest of the counting and bijection arguments then close.

Section 8.1, property (iv): The printed inverse can fail to produce strictly increasing coordinates

Journal pages 503–504 · property (iv) immediately before Proposition 8.1

Incorrect as written · verified repair

The version of record assigns an increment $1/(n+1)$ whenever a new zero-indexed letter appears. For $n=3$, $m=1$, the valid annotated sketch

$$\alpha_1^{(0)} \alpha_2^{(0)} \alpha_1^{(1)} \alpha_3^{(0)} \alpha_2^{(1)} \alpha_3^{(1)}$$

yields $z_1 = 1/4$, $z_2 = 1/2$, $z_3 = 5/4$, $z_4 = 3/2$, and $z_5 = 3/2$, contradicting the required strict order $z_1 < \dots < z_6$. ArXiv:1604.06554v4 gives the verified repair to that branch: with $z_0 = 0$, set $z_p = z_{p-1} + 2^{-p}$ for a letter $\alpha_i^{(0)}$, while retaining the version of record’s rule $z_p = x_i + s$ for $s \neq 0$. The tail of the decreasing geometric series is smaller than the next integer displacement, so all prescribed comparisons remain strict and the annotated order is realized.

Source: Bernardi, corrected arXiv:1604.06554v4, author-copy pages 34–35

Theorem 8.8 proof chain: Surjectivity and the transitive bijectivity upgrade are complete after repair

Journal pages 512–513 · Lemmas 8.12–8.13 and proof of Theorem 8.8

Correct and complete after verified repair

An S -move crosses exactly one Catalan wall absent from A_S , so equivalence classes are precisely the A_S -regions and each finite class has one lexicographic maximum. Proposition 8.1 translates every improving move into the forbidden cadet condition, proving Lemma 8.13. Every global maximum is local, which gives surjectivity without transitivity. For transitive S , Theorem 4.6 supplies equal cardinalities; its proof-critical inputs are Zaslavsky’s region-count formula and the exponential/tree-path tools cited to Stanley’s Enumerative Combinatorics, Volume 2. Hence the final bijectivity step is justified.

Sections 3–7: The enumerative proof is internally consistent

Journal pages 472–502 · signed involutions, coboundary generating functions, and their proofs

Correct and complete

The cadet-sequence definitions match the forbidden-intercept conditions in each orientation, the sign-reversing involutions pair exactly the non-transitive tree contributions, and the coefficient extractions from the multitype series recover the fixed-dimension arrangements. The graph and tree encodings are reversible, and the energy statistic reproduces the coboundary weight. No additional unresolved proof gap was found in the full manuscript.

Novelty

No non-novelty findings No non-novelty findings.

METHOD

Exact-source disclosure

This audit evaluated only the linked journal version of record. An arXiv preprint, accepted manuscript, or later correction is not silently substituted for that source.

Sources checked

1. [Version of record](#) · [Advances in Mathematics 335 \(2018\), 466–518](#)
2. [Exact](#) [Advances in Mathematics](#) version of record
3. [Corrected arXiv version 4](#)
4. [Bernardi, corrected arXiv:1604.06554v4, author-copy pages 34–35](#)

Audit instructions

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.
2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.
3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.
4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.
5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.
6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.
7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.
8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.
9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.
10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.
11. Write every mathematical expression as valid, compilable LaTeX. Use $\dots$ for inline mathematics and $\dots$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.
12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT