

Correctness, proof completeness, novelty, and source audit

Twisted Diophantine Approximation II: Uniform Theory

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Paper	arXiv:2609.36412v1
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Prompt	Standardized prompt
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Paper license	CC BY 4.0
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A green 'no issues found' label may coexist with yellow suspected typos or minor formal issues that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	No statement issues found
02	Proofs	No proof issues found
03	Novelty	No novelty concerns found
04	Sources	No source-use issues found

This independent report links to the source paper on arXiv; it does not reproduce or redistribute that paper.

01 / STATEMENTS

Statements

No statement issues found

The pressure formulae for the lower and upper endpoint sets, the no-dimension-drop identity, the near-power and uniform-exponent formulae, the Hausdorff-measure zero-full law, the one-dimensional continued-fraction formulae, the escape-of-mass estimates and sharpness statement, and the transference applications are correctly stated in arXiv:2609.36412v1. The exceptional power set and every required regularity or nonemptiness qualification are carried into the conclusions. No mathematical statement requiring correction was found.

Theorem 1.2: The pressure formula and no-dimension-drop identities have the stated scope

PDF pages 5–6 and 31–39 · Theorem 1.2, Theorem 5.1, Proposition 5.3, and proof · arXiv:2609.36412v1

No issue found

For a continuous strictly decreasing lower-quasimultiplicative function, the equal-radius mesh gives uniform upper and lower estimates for every constant multiple. Taking the infimum and supremum in the constant and then letting the mesh parameter tend to infinity produces the two endpoint pressures. The variable-scale Cantor construction supplies a single function $\vartheta = o(\psi)$ whose dimension approaches the lower pressure, so the intersection defining $\text{Sing}_A(\psi)$ loses no dimension. The union identity is countable after restricting constants to integers. These facts also prove that constant-rescaling invariance is equivalent to equality of the two pressures.

Theorem 1.4; Proposition 1.6; Corollary 1.7: Near-power and uniform-exponent dimensions include the critical qualifications

PDF pages 7–8 and 39–48 · Section 6 · arXiv:2609.36412v1

No issue found

Away from the finite set $\mathcal{E}_{m,n} = \{b/a : 1 \leq a \leq m, 1 \leq b \leq n\}$, threshold crossings of the normalized successive minima have zero density. This makes the pressure independent of the constant and converts it into the stated rank integral. Comparison with nearby powers proves the same formula for functions of limiting power order. At every exponent, including critical ones, monotonicity of the approximation sets and the no-drop construction give the displayed superlevel, strict-superlevel, and conditional exact-level formulae.

Theorem 1.9: The zero-full alternatives are correctly formulated on the orbit-closure group

PDF pages 9–10 and 48–51 · Theorem 1.9 and proof · arXiv:2609.36412v1

No issue found

Every approximable shift lies in $H_A = \overline{A\mathbb{Z}^n \bmod \mathbb{Z}^m}$, and the proof works with its intrinsic Haar and Hausdorff measures. Translation by the dense subgroup amplifies any positive finite Hausdorff measure to a Haar-conull invariant set; infinite measure gives local fullness directly. Difference and sum constructions then establish exact amplification and show that the two endpoint sets are Borel subgroups. The zero-dimensional finite-group case is separated explicitly.

Propositions 1.10–1.11: The reversed endpoint representations and dimension continuity are valid

PDF pages 10–11 and 51–53 · Propositions 1.10–1.11 and proofs · arXiv:2609.36412v1

No issue found

Sparse piecewise-linear interpolation constructs a single smaller function for each point in the singular endpoint and a single larger obstruction for each point outside the upper endpoint. The diagonal pressure construction can be carried out within the lower-quasimultiplicative class, which gives the stated supremum and infimum identities for the endpoint dimensions rather than only the set-theoretic union and intersection formulas.

Theorem 2.2: The one-dimensional formulae correctly specialize the general pressure

PDF pages 13–15 and 53–62 · Theorem 2.2 and proof · arXiv:2609.36412v1

No issue found

The continued-fraction profile identifies the two active branches of the lattice count and isolates the convergent indices at which they cross the approximation curve. Logarithmic sparsity makes discarded transition intervals negligible. Telescoping the remaining pressure increments produces the two printed products in the slow and fast regimes, including the finite-index cases of dimensions one and zero. Multiplying or argument-rescaling ψ shifts the profile by only a bounded amount and leaves the dimension unchanged.

Propositions 2.4–2.5 and 2.7: The escape-of-mass upper bounds and sharpness conclusion are correct

PDF pages 16–17 and 62–69 · Propositions 2.4–2.5, 2.7, and proofs · arXiv:2609.36412v1

No issue found

The rank integral is bounded by the time spent below a first-minimum cutoff and by the product of all active minima. At the critical exponent, Minkowski’s product control leaves at most $d - 1$ active factors and yields the improved coefficient $m(d - 1)/d$. For sharpness, the tent template has active proportions $\eta m/d$ and $\eta n/d$, while the zero phase has proportion $1 - \eta$; the template-realization theorem produces a matrix with exactly that escape proportion. The pressure calculation and monotonicity in the approximation constant give the matching lower bound, so every $\eta \in [0, 1]$ occurs.

Propositions 2.8–2.10: The transference and trivial-shift conclusions preserve all quantifiers

PDF pages 17–18 and 69–71 · Propositions 2.8–2.10 and proofs · arXiv:2609.36412v1

No issue found

A sequence of good dual approximants confines the uniformly approximable shifts to character strips of measure $2d\delta$, and the zero-full theorem turns positive measure at a smaller constant into the contradiction needed for nullity. Conversely, the finite-scale transference lemma with $\kappa_d = 2^{1-d}(d!)^2$ produces an admissible denominator for every shift when the transpose is not approximable. Adding a fixed positive vector gives the positive-denominator variant with the factor three. The telescoping difference argument for two denominators proves that sufficiently small approximation sets contain exactly the prescribed orbit shifts.

02 / PROOFS

Proofs

No proof issues found

All central proof chains close in the exact source. The audit followed the lattice-count comparison into both covering constructions, the pressure and variable-scale diagonal arguments, the normalized-minima analysis for powers, the compact-group zero-full argument, the continued-fraction specialization, the escape-of-mass template construction, and both transference directions. Constants, directions of inclusions, limiting operations, and exceptional cases agree with the stated conclusions. No incorrect or materially incomplete proof was found.

Lemmas 4.1–4.4: The successive-minima profile gives the required lattice counts

PDF pages 22–24 · Section 4.1 · arXiv:2609.36412v1

No issue found in the reviewed proof

After conjugating the (R, Q) box to a cube, the companion-paper count is exactly the product defining $\mathcal{N}_A(R, Q)$ up to a dimension-only constant. Fixed dilations preserve the count up to such a constant, and translated boxes contain no more lattice points than the corresponding centered difference box. The monotonicity and scaling estimates used later follow with the displayed exponents.

Propositions 3.1–3.2: The upper and lower covering constructions have matching transition counts

PDF pages 20–31 · Sections 3–4 · arXiv:2609.36412v1

No issue found in the reviewed proof

The upper tree uses maximal disjoint families whose triples cover every omitted ball; the containment margins place all relevant lifted centers in the numerator box, while separation supplies the denominator count. The lower tree reserves both approximation and denominator margins, giving the reverse transition estimate. The condition $\psi(T_j) \leq \rho\psi(T_{j+1})$ bridges every parameter between mesh points, and the mass distribution bound controls arbitrary intermediate radii rather than only construction scales.

Theorem 5.1 and Proposition 5.3: Uniform pressure estimates and the no-drop diagonalization close

PDF pages 31–39 · Section 5 · arXiv:2609.36412v1

No issue found in the reviewed proof

Lower quasimultiplicativity makes consecutive denominator bounds and radii satisfy the construction hypotheses uniformly once L is large. The fixed branching constants contribute only $O(L^{-1})$, uniformly in the rescaling constant. For no drop, the block lengths dominate every preceding transition while their excess lengths are sublinear; the interpolated function remains decreasing and lower quasimultiplicative, is $o(\psi)$, and retains the pressure lower bound after the transition blocks are removed.

Section 6: The near-power pressure and exponent-level arguments handle all threshold cases

PDF pages 39–48 · Lemmas 6.1–6.6 and Proposition 1.6 · arXiv:2609.36412v1

No issue found in the reviewed proof

Exterior-power covolumes control sums of logarithmic minima. When an exponent is outside $\mathcal{E}_{m,n}$, a separated-block argument shows that every zero threshold has density zero. On the remaining mesh rectangles, the active exterior subgroup and vertical slope are constant, so the pressure telescopes to the rank identity with an error uniform in the constant. Order comparison with neighboring powers gives both one-sided exponent limits; the variable-power no-drop lemma supplies the exact-level lower bound whenever that level is nonempty.

Theorem 1.9 and Propositions 1.10–1.11: The invariant-measure and endpoint interpolation arguments are complete

PDF pages 48–53 · Section 7 · arXiv:2609.36412v1

No issue found in the reviewed proof

The proof treats separately zero, finite positive, and infinite Hausdorff measure. In the finite case, dense-subgroup invariance extends to H_A and uniqueness of Haar measure makes the invariant set conull; comparison of the dimension function with the intrinsic power measure supplies local fullness. Translation and subtraction of witnesses give the factors used in exact amplification and subgroup closure. The sparse interpolation sequences used for the endpoint identities preserve monotonicity and the required little- o relations.

Theorem 2.2: The continued-fraction decomposition and telescoping products agree

PDF pages 53–62 · Section 8 · arXiv:2609.36412v1

No issue found in the reviewed proof

Successive convergents determine the first lattice minimum on each branch interval, and unimodularity supplies the second. The relevant crossing times are ordered by the slow- or fast-decay hypothesis. Boundary blocks have zero logarithmic density, while the retained intervals contribute exactly the numerator factors appearing in formulas (2.7) and (2.10). The pressure normalization matches the final denominator $-\log \psi(Y_{k_i})$.

Propositions 2.4–2.5 and 2.7: The rank bounds and sharp template calculation give the advertised coefficient

PDF pages 62–69 · Section 9 · arXiv:2609.36412v1

No issue found in the reviewed proof

The profile product identity separates the rank term from the normalized lattice-count term. Shrinking the cutoff and integrating yields the general upper bound; at the critical exponent the first-minimum escape set controls every active factor and produces $m(d-1)/d$. In the sharp template, cycle count and unfinished-cycle length are sublinear, so both escape frequency and pressure are determined by the three printed phase proportions. For constants outside the initially normalized range, the zero-phase contribution is nonnegative and monotonicity of the approximation sets gives the same endpoint lower bound.

Propositions 2.8–2.10: The dual-strip, full-space, and telescoping arguments are valid

PDF pages 69–71 · Section 10 · arXiv:2609.36412v1

No issue found in the reviewed proof

The scalar-product identity converts simultaneous primal and dual inequalities into a strip condition with the correct $m + n$ factor. Haar pushforward gives its exact measure, and Fatou’s inequality in the required direction bounds the $\liminf$. The imported finite-scale transference lemma is applied with the dual inverse functions at matching parameters. For trivial shifts, two eventual witnesses subtract to a homogeneous approximation that the hypothesis forbids unless the denominator difference is zero, forcing a single orbit point.

03 / NOVELTY

Novelty

No novelty concerns found

No non-novelty findings.

04 / SOURCES

Sources

No source-use issues found

Every direct proof-critical citation was located and checked in an exact public version or a disclosed proof-bearing counterpart. The lattice-count input from the companion paper agrees with Henk’s two underlying results; the mass-distribution, compact-group, template-realization, Haar-pushforward, and inhomogeneous-transference inputs all have the hypotheses and conclusions required by the focal proof. Recursive non-book dependencies are represented in the graph, and the three published books are explicit terminal sources.

Companion Lemma 5.1 and Henk’s lattice-point bounds: The counting input matches the focal profile exactly

Focal PDF pages 22–23 · companion arXiv:2609.36411v1 pages 14–15 · Henk Lemma 2.1 and Theorem 1.5 · arXiv:2609.36412v1

Applicable and sufficient

The companion lemma compares the centered (R, Q) box count with $\prod_i \max\{1, R/\lambda_i\}$ after the focal diagonal normalization. Henk Lemma 2.1 yields the fixed-dilation comparison through lattice cosets, and Theorem 1.5 yields the required upper product. The companion paper proves the matching lower bound internally by taking bounded integer combinations of linearly independent minimizing vectors.

Verifiable source: Kim and Neckrasov, arXiv:2609.36411v1

Beresnevich–Velani Mass Distribution Principle: The cited principle directly proves the lower-dimension step

Focal PDF pages 27–30 · source Annals page 975 · arXiv:2609.36412v1

Applicable and sufficient

The source states the principle for a probability measure supported on a metric-space subset and proves it by applying the ball estimate to an arbitrary fine cover. Taking $f(r) = r^s$ gives exactly the focal Lemma 4.8, with no Euclidean-only hypothesis missing.

Verifiable source: Beresnevich and Velani, *Annals of Mathematics* 164 (2006), 971–992

Kleinbock–Shah–Starkov and Lee: The compact-group inputs support the zero-full proof

Focal PDF pages 9 and 48–49 · KSS Lemma 1.1.3 · Lee Theorem 20.12 · arXiv:2609.36412v1

Applicable and sufficient

Lee’s Closed Subgroup Theorem makes H_A a compact Lie subgroup of the torus. KSS Lemma 1.1.3 says that a finite measure’s stabilizer is closed; continuity of translation on measures proves the assertion immediately. Since Γ_A is dense in H_A , these facts justify the extension from discrete translation invariance to Haar invariance used in Theorem 1.9.

Verifiable source: Kleinbock, Shah, and Starkov, *Handbook of Dynamical Systems* 1A (2002)

Das–Fishman–Simmons–Urbański template realization: The realization theorem covers the sharpness template

Focal PDF pages 65–67 · source Definition 4.1 and Theorem 4.2 · arXiv:2609.36412v1

Applicable and sufficient

The focal tent functions are ordered, balanced, have allowed coordinate slopes, and satisfy the convexity and quantized-slope rule at their only strict gap. The source theorem realizes every such template within bounded error as the successive-minima function of a matrix; its proof includes the zero-average-contraction case relevant here. This is precisely the existence statement needed for Proposition 2.5. The checked source is the final public preprint arXiv:1901.06602v13 corresponding to the published article; a separate publisher PDF was not checked.

Verifiable source: Das, Fishman, Simmons, and Urbański, final public preprint arXiv:1901.06602v13

Halmos §63, Theorem C: Haar pushforward gives the exact character-strip measure

Focal PDF page 69 · Lemma 10.1 · arXiv:2609.36412v1

Applicable and sufficient

For every nonzero integer vector $\mathbf{p}$, the character $\mathbb{T}^m \rightarrow \mathbb{T}$, $\mathbf{b} \mapsto \mathbf{p} \cdot \mathbf{b}$, is a continuous surjective homomorphism. Halmos’ theorem identifies the pushforward of normalized Haar measure with normalized Haar measure on the circle, so the inverse image of the interval $[-d\delta, d\delta]$ has measure $2d\delta$ when $\delta < 1/(2d)$.

Verifiable source: Paul R. Halmos, *Measure Theory*, §63

Moshchevitin–Neckrasov Lemma I: The transference constant and normalization agree

Focal PDF pages 70–71 · source arXiv:2503.21180v3 page 28 · arXiv:2609.36412v1

Applicable and sufficient

Lemma I assumes a uniform lower bound for nonzero transpose vectors of norm at most S and produces, for every shift, a primal denominator of norm at most T with error at most κ_d/S . The focal variables are substituted in the same order and use the same constant $\kappa_d = 2^{1-d}(d!)^2$. The source explicitly derives this lemma from Bugeaud–Laurent Lemma 3, whose proof invokes Cassels Chapter V, Theorem XVII(B); that recursive chain is represented in the graph.

Verifiable source: Moshchevitin and Neckrasov, arXiv:2503.21180v3

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Suspected typographical errors and minor formal issues remain fully visible in yellow with their locations and proposed repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs result from green (no issues found).

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: [arXiv:2609.36412v1](#)
2. Kim and Neckrasov, [arXiv:2609.36411v1](#)
3. Beresnevich and Velani, *Annals of Mathematics* 164 (2006), 971–992
4. Kleinbock, Shah, and Starkov, *Handbook of Dynamical Systems* 1A (2002)
5. Das, Fishman, Simmons, and Urbański, final public preprint [arXiv:1901.06602v13](#)
6. Paul R. Halmos, *Measure Theory*, §63
7. Moshchevitin and Neckrasov, [arXiv:2503.21180v3](#)

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Do not investigate, infer, or comment on authorship, contribution credit, acknowledgments, changes in author lists between versions or sources, collaboration history, or research conduct. These matters are outside the audit and must not appear in any finding, summary, source assessment, or report text. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only comparison of central mathematical results with earlier papers. First identify only the central results that the reviewed paper genuinely presents as new contributions. If the manuscript is positioned as a report, review, survey, expository article, lecture notes, or a similar work, do not assess results that it presents as earlier, known, surveyed, or expository material; assess only any central results that it explicitly claims as new.

A qualifying earlier paper must be a publicly available journal article, proceedings article, or stable public preprint that states and supports the result in written form. Report a result only when such a paper, publicly available before the earliest public version of the reviewed paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from a theorem in that paper.

Talks, lectures, seminar or conference presentations, slides, posters, videos, abstracts, announcements, personal communications, theses or dissertations, and other non-paper materials do not establish non-novelty, even if they state or outline the result. They may be used only to locate a qualifying earlier paper and must not be mentioned in the report.

For each non-novelty finding, provide the qualifying earlier paper, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later paper cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the reviewed paper itself explicitly presents as known, a survey item, or an alternative proof.

Novelty findings concern mathematical statements only. Do not use the novelty check to discuss author-level priority, attribution, authorship or contribution provenance, or changes in a work's author list.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, add statement-level "no prior source found" entries, or mention excluded source types or other suppressed observations even as context. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use $\$ \dots \$$ for inline mathematics and $\$ \$ \dots \$ \$$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- *Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.*
- *Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.*
- *Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.*

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Authorship, contribution credit, acknowledgments, author-list changes, collaboration history, and authorship or contribution provenance remain outside scope and must not appear in the Sources assessment. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.