

Correctness, proof completeness, novelty, and source audit

Twisted Diophantine Approximation I: Asymptotic Theory

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Paper	arXiv:2609.36411v1
Source version	Submitted 29 September 2026; announced September 2026.
Audit date	October 23, 2026
Prompt	Standardized prompt
Novelty cutoff	September 29, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A green 'no issues found' label may coexist with yellow suspected typos or minor formal issues that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	No statement issues found
02	Proofs	No proof issues found
03	Novelty	No novelty concerns found
04	Sources	No source-use issues found

This independent report links to the source paper on arXiv; it does not reproduce or redistribute that paper.

01 / STATEMENTS

Statements

No statement issues found

The exact zero-one law, its successive-minima form, the zero-infinity and critical bad-target classifications, the nonsingular consequence, the Hausdorff-measure and power-law dimension results, and the scalar equivalence with the Fuchs-Kim criterion are correct under the printed dyadic-regularity hypotheses.

Theorem 1.1 and Corollary 1.4: The fixed-matrix zero-one criterion and its successive-minima form are correct

PDF pages 4-5 and 14-28 · Theorem 1.1, Corollary 1.4, Lemmas 5.1-5.3, Proposition 6.2, and Proposition 7.3 · arXiv:2609.36411v1

No issue found

For every fixed real matrix and every non-increasing dyadically regular approximation function, the proof identifies the summand $M_A(\psi(2^k), 2^k)$ with the measure of a disjoint Vitali packet up to constants. Convergence follows from a direct cover. In the divergent case the mixed-scale estimate, fixed-sublattice summation, and incidence bound give quasi-independence on average; the generalized Borel-Cantelli inequality gives positive measure and the independently proved fixed-matrix zero-one law upgrades it to full measure. Rescaling the mixed box to a cube gives the stated product of truncated successive minima with the correct parameters R_k and t_k .

Verifiable source: [Exact manuscript](#), [arXiv:2609.36411v1](#)

Corollary 2.1 and Proposition 2.2: The Lagrange zero-infinity law and critical classification follow with the stated quantifiers

PDF pages 9-10 · Corollary 2.1 and Proposition 2.2 · arXiv:2609.36411v1

No issue found

Lemma 5.3 makes the series criterion invariant under every fixed positive rescaling of ψ . Countable intersections of the divergent full-measure sets force the normalized liminf to be zero, while the complements of all convergent rescalings force it to be infinite. At $\psi(q) = q^{-n/m}$, the flow normalization gives $R_k = 1$ and $t_k = nk \log 2$, so the series is comparable term by term to $\Delta_k(A)$ and the null-or-conull conclusions for all three bad-target sets follow.

Verifiable source: [Exact manuscript](#), [arXiv:2609.36411v1](#)

Corollary 2.4: Nonsingularity forces divergence of the critical series

PDF page 10 · Corollary 2.4 · arXiv:2609.36411v1

No issue found

Dani's correspondence identifies nonsingularity with failure of forward-orbit divergence. Hence the orbit returns to a compact set at arbitrarily large times. Mahler compactness gives a uniform positive lower bound for its first minimum on those returns, and bounded distortion over the fixed sampling interval $n \log 2$ transfers that bound to infinitely many sampled times. Each corresponding $\Delta_k(A)$ is bounded below by a fixed positive constant, so the critical series diverges.

Verifiable source: [Dani](#), [exact 1985 version of record](#)

Corollaries 3.1 and 3.2: The Hausdorff criteria and power-law dimension formula are correct

PDF pages 11-12 · Corollaries 3.1 and 3.2 · arXiv:2609.36411v1

No issue found

The convergence implication uses the same Vitali cover as Theorem 1.1 with the s -cost of each selected ball. For divergence, applying Theorem 1.1 to $\varphi = \psi^{s/m}$ makes the enlarged-ball limsup conull, and the general mass transference principle on the flat torus yields infinite s -dimensional Hausdorff measure for the original limsup. Substitution of $\psi_v(q) = q^{-v}$ then gives the lower bound at $s = n/v$ and the elementary convergent cover gives every upper bound $s' > n/v$, proving $\dim_H W_A(\psi_v) = \min\{m, n/v\}$ under the displayed critical-series hypothesis.

Verifiable source: [Beresnevich-Velani mass transference source](#)

Proposition 4.1: The new dyadic series is equivalent to the scalar Fuchs-Kim series

PDF pages 12-14 · Proposition 4.1 · arXiv:2609.36411v1

No issue found

For $q_s \leq 2^j < q_{s+1}$, best-approximation separation gives the required upper bound on the homogeneous cluster count and hence a lower bound for M_α . In the reverse direction, the explicit multiples of q_s counted in Beresnevich-Haynes-Velani equation (6.9) give the required lower cluster count. Fuchs-Kim Lemma 2.1 supplies the summability of $q_s \psi(q_s)$ from convergence of the block-minimum series, and dyadic regularity sums the remaining terms geometrically. Both implications of the displayed convergence equivalence follow.

Verifiable source: [Fuchs-Kim, exact arXiv:1501.04714v1](#)

02 / PROOFS

Proofs

No proof issues found

The central proofs are correct and complete. The lattice-counting normalization, fixed-matrix zero-one decomposition, Vitali cardinality estimates, mixed-scale packing, exterior-product formula, fixed-sublattice geometric summation, bounded-incidence argument, and all application deductions close with their printed hypotheses. Every proof-critical cited input checked below is used in its valid range.

Sections 5 and 7: The lattice counts and Vitali mass estimates are correct and complete

PDF pages 14-21 · Lemmas 5.1-5.3 and 7.1-7.2 and the convergence proof · arXiv:2609.36411v1

No issue found in the reviewed proof

Henk's sublattice comparison gives fixed-dilation control, and his successive-minima upper bound combines with the manuscript's independent-vector lower bound to give the two-sided cube count. The diagonal rescaling is algebraically exact. In Lemma 7.1, the lower Vitali count follows by charging orbit points to selected triple balls, while the upper count uses disjoint translated lattice boxes inside one ambient box; both comparisons preserve uniform constants. These estimates give the Lebesgue and Hausdorff convergence arguments and the correct one-scale masses for divergence.

Verifiable source: [Henk, Successive Minima and Lattice Points, arXiv:math/0204158v1](#)

Section 6: The fixed-matrix zero-one proof handles every escaping-coordinate pattern

PDF pages 16-18 · Lemma 6.1 and Proposition 6.2 · arXiv:2609.36411v1

No issue found in the reviewed proof

Every infinite sequence of distinct integer solutions has a subsequence whose coordinates are either fixed or escape with fixed signs, yielding a countable decomposition. For a component pattern, all centers lie in a coset of the closed subgroup generated by the escaping columns. A proper coset is null; if the subgroup is dense, decrementing one escaping coordinate gives one-sided invariance under each generator. Translation invariance turns those inclusions into invariance modulo null sets, and the Fourier argument forces the component event to be null or conull. The countable union therefore has measure zero or one.

Verifiable source: [Exact manuscript, arXiv:2609.36411v1](#)

Sections 8 and 9: The quasi-independence-on-average argument closes at every rank

PDF pages 22-28 · Lemmas 8.1-8.2, 9.1-9.3, and proof of Proposition 7.3 · arXiv:2609.36411v1

No issue found in the reviewed proof

The local packing lemma bounds later selected balls meeting a fixed earlier ball by a ratio of mixed and one-scale lattice counts. Lemma 9.1 correctly converts each mixed mass to the largest exterior-coordinate term of its active primitive sublattice. For a fixed proper sublattice, the two ordinary geometric arms and the two residual endpoint arms partition all pairs and charge only endpoint masses. Nested mixed boxes form two sublattice chains at each endpoint; equal ranks force equal spans and hence equal primitive sublattices, so at most $2d$ proper sublattices charge one index. Full-rank pairs contribute quadratically and proper-rank pairs linearly, giving Proposition 7.3 and completing divergence.

Verifiable source: [Exact manuscript, arXiv:2609.36411v1](#)

Parts I and II source applications: The Dani, mass-transference, scalar-counting, and Borel-Cantelli inputs are applied correctly

PDF pages 5, 10-14, and 20-21 · Remark 1.5, Corollaries 2.4 and 3.1, Proposition 4.1, Lemma 7.2, and the divergence proof · arXiv:2609.36411v1

No issue found in the reviewed proof

Dani Theorems 2.14 and 2.20 have the required divergence and bounded-orbit correspondences. The general mass transference theorem applies to the torus with $g(r) = r^m$ and $f(r) = r^s$. Fuchs-Kim's convergence estimate and Beresnevich-Haynes-Velani's elementary count have exactly the scalar hypotheses used. The generalized Borel-Cantelli lower bound and Hausdorff-Cantelli covering lemma are invoked in their standard probability and metric-measure ranges. No proof depends on a stronger conclusion than its source provides.

Verifiable source: [Exact source bundle for the focal proof chain](#)

03 / NOVELTY

Novelty

No novelty concerns found

No non-novelty findings.

04 / SOURCES

Sources

No source-use issues found

Every proof-critical external input was located in an exact public version or published source and is sufficient for the use made of it. Henk supplies both lattice-count estimates and a proof of Minkowski's second theorem; Dani supplies the two dynamical correspondences; Beresnevich-Velani supplies the general mass transference theorem; Fuchs-Kim and Beresnevich-Haynes-Velani supply the scalar estimates; and the two cited books supply the probability and Hausdorff covering lemmas. Mahler compactness is represented by a standard terminal book source in the dependency graph.

Henk Lemma 2.1, Theorems 1.3 and 1.5: The lattice-counting source gives exactly the required comparisons

Focal PDF pages 14-15 and 24-26 · Lemmas 5.1 and 9.1 · source [arXiv:math/0204158v1](#) · audited focal [arXiv:2609.36411v1](#)

Applicable and sufficient

Henk Lemma 2.1 compares the lattice-point count of a convex body with the count in a fixed-index sublattice and yields the focal fixed-dilation bound when the sublattice is $\lceil 2C \rceil \Lambda$. Henk Theorem 1.5 gives the product upper bound in terms of successive minima, while the focal paper supplies its own lower bound. Henk Theorem 1.3 states Minkowski's second theorem, and Section 3 gives the proof used for the covolume-product comparison in Lemma 9.1. The exact source archive and PDF were checked.

Verifiable source: [Henk](#), [arXiv:math/0204158v1](#)

Dani Theorems 2.14 and 2.20: The exact published proofs support both orbit correspondences

Focal PDF pages 5 and 10 · Remark 1.5 and Corollary 2.4 · Dani printed pages 63-65 · audited focal [arXiv:2609.36411v1](#)

Applicable and sufficient

The exact 1985 version-of-record scan was inspected. Theorem 2.14 proves that singular systems are precisely those whose associated forward diagonal trajectories diverge, and Theorem 2.20 proves the bounded-trajectory characterization of badly approximable systems. The focal block convention differs only by the standard block permutation and transpose convention. The journal's correction notice does not amend either theorem.

Verifiable source: [Dani](#), [version of record at EuDML](#)

Beresnevich-Velani Section 6.1, Theorem 3: The general mass transference theorem applies on the flat torus

Focal PDF page 11 · proof of Corollary 3.1 · source Section 6.1, Theorem 3 · audited focal [arXiv:2609.36411v1](#)

Applicable and sufficient

The source theorem is stated for a locally compact metric space with a doubling dimension function and comparable ball measure. The torus with the supremum metric has these properties for $g(r) = r^m$. With $f(r) = r^s$, the required quotient is monotonic and the transformed radius is $r^{s/m}$, exactly the focal enlarged target. The radii tend to zero and Theorem 1.1 supplies conullity of the enlarged limsup, so every source hypothesis is met.

Verifiable source: [Beresnevich and Velani](#), [Annals of Mathematics 164 \(2006\)](#)

Fuchs-Kim Lemma 2.1 and Beresnevich-Haynes-Velani equation (6.9): The scalar convergence and homogeneous-counting inputs are exact

Focal PDF pages 12-14 · Proposition 4.1 · source arXiv:1501.04714v1, Lemma 2.1 proof, and arXiv:1511.06862v2, Section 6.1 equation (6.9) · audited focal arXiv:2609.36411v1

Applicable and sufficient

In the Fuchs-Kim proof, convergence of the block-minimum series rules out the large-endpoint case and gives $q_s \psi(q_s)$ bounded by two adjacent block sums, hence summability over s . Beresnevich-Haynes-Velani equation (6.9) counts the positive multiples tq_s satisfying both $tq_s \leq N$ and $t\|q_s\alpha\| \leq \varepsilon$; adjoining zero and negative multiples gives precisely the focal lower bound. The continued-fraction inequality $q_{s+1}\|q_s\alpha\| \geq 1/2$ then produces the printed comparison.

Verifiable source: [Beresnevich-Haynes-Velani, exact arXiv:1511.06862v2](#)

Sprindzuk Lemma 5 and Bernik-Dodson Lemma 3.10: The terminal book lemmas have exactly the strengths used

Focal PDF pages 20-21 · convergence proof, Lemma 7.2, and divergent-case proof · arXiv:2609.36411v1

Applicable and sufficient

Sprindzuk's second-moment form of the divergent Borel-Cantelli lemma gives the displayed lower bound for the limsup measure from the finite intersection sums. Bernik-Dodson's Hausdorff-Cantelli lemma gives zero Hausdorff measure when the sum of the covering diameters to the power s converges. The focal probability space is normalized Lebesgue measure on the torus, and the selected covering diameters tend to zero, so both applications meet their stated hypotheses. These published books are terminal under the dependency-graph stopping rule.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Suspected typographical errors and minor formal issues remain fully visible in yellow with their locations and proposed repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs result from green (no issues found).

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. [Exact source bundle for the focal proof chain](#)
2. [Exact manuscript, arXiv:2609.36411v1](#)
3. [Dani, version of record at EuDML](#)
4. [Beresnevich and Velani, Annals of Mathematics 164 \(2006\)](#)
5. [Fuchs-Kim, exact arXiv:1501.04714v1](#)
6. [Henk, arXiv:math/0204158v1](#)
7. [Beresnevich-Haynes-Velani, exact arXiv:1511.06862v2](#)

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Do not investigate, infer, or comment on authorship, contribution credit, acknowledgments, changes in author lists between versions or sources, collaboration history, or research conduct. These matters are outside the audit and must not appear in any finding, summary, source assessment, or report text. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only comparison of central mathematical results with earlier papers. First identify only the central results that the reviewed paper genuinely presents as new contributions. If the manuscript is positioned as a report, review, survey, expository article, lecture notes, or a similar work, do not assess results that it presents as earlier, known, surveyed, or expository material; assess only any central results that it explicitly claims as new.

A qualifying earlier paper must be a publicly available journal article, proceedings article, or stable public preprint that states and supports the result in written form. Report a result only when such a paper, publicly available before the earliest public version of the reviewed paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from a theorem in that paper.

Talks, lectures, seminar or conference presentations, slides, posters, videos, abstracts, announcements, personal communications, theses or dissertations, and other non-paper materials do not establish non-novelty, even if they state or outline the result. They may be used only to locate a qualifying earlier paper and must not be mentioned in the report.

For each non-novelty finding, provide the qualifying earlier paper, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later paper cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the reviewed paper itself explicitly presents as known, a survey item, or an alternative proof.

Novelty findings concern mathematical statements only. Do not use the novelty check to discuss author-level priority, attribution, authorship or contribution provenance, or changes in a work's author list.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, add statement-level "no prior source found" entries, or mention excluded source types or other suppressed observations even as context. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use $\$ \dots \$$ for inline mathematics and $\$ \$ \dots \$ \$$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- *Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.*
- *Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.*
- *Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.*

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Authorship, contribution credit, acknowledgments, author-list changes, collaboration history, and authorship or contribution provenance remain outside scope and must not appear in the Sources assessment. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.