MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On integers that are representable as the sum of two units

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Paper	arXiv:2608.15903v1
Source version	Submitted 16 August, 2026; announced August 2026.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	August 16, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains unverified proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The effective height theorem and its odd-degree and primitive-field consequences are correct. The exact quintic classification into 255 equivalence classes and eleven represented positive integers is not independently verifiable from the manuscript because its exhaustive computational steps are reported without reproducible certificates or code.

Theorem 1: The nonconjugacy hypothesis gives an effective uniform height bound

Pages 1–2 and 6–12 · Theorem 1, Lemmas 5–6, Propositions 1–2 · arXiv:2608.15903v1

Correct

After adjoining the two units to a common subfield, the logarithmic embedding and regulator minors bound their unit exponents by the largest conjugate. For every four-term conjugate relation, Lemma 6 combines unit matching with Matveev's lower bound to give either a controlled second-smallest term or a bounded exponent. Proposition 1 converts these local alternatives into a global bound, and Lemma 2 makes it effective. The hypothesis of Theorem 1 excludes the sole nontrivial exceptional case in which the two units are conjugate; rational units are immediately bounded. This proves the stated bound for $\log|n|$, $h(\varepsilon)$, and $h(\delta)$.

Corollary 1: The Galois and primitivity conditions exclude every conjugate-unit obstruction

Pages 2 and 12–13 · Corollary 1 and Lemmas 7–8 · arXiv:2608.15903v1

Correct

If $n-\varepsilon$ is a distinct conjugate of ε, the embeddings pair into blocks preserved by the normal-closure Galois group. Hence the degree is even and the transitive group lies in $C_2 \wr S_{d/2}$. Equivalently, shifting by $n/2$ makes the minimal polynomial even and produces a proper subfield of index two. The corollary's hypothesis rules this out in every subfield. Odd-degree fields and primitive fields of degree at least three therefore satisfy Theorem 1 and admit an effective finite search.

Theorem 2: The exhaustive quintic classification is not independently certified

Pages 2–3 and 13–20 · Theorem 2, Section 5, and Tables 2–4 · arXiv:2608.15903v1

Not able to verify

The theorem claims exactly 255 non-equivalent unit pairs and

$$N_K = \{1, 2, 3, 4, 5, 6, 7, 10, 11, 15, 251\}$$

for the field defined by $X^5 - 5X^3 - X^2 + 5X + 1$. The theoretical argument reduces the problem to finite computations, and Table 4 supplies explicit positive witnesses. Exhaustiveness, however, depends on seven high-precision LLL reductions, a test of all 1,676,902,296 exponent quadruples, and complete Magma solutions of 1000 unit equations. The manuscript supplies neither the reduced bases and rigorous rounding intervals nor code, logs, or checkable certificates for these runs. No contradiction was found, but the negative and counting assertions cannot be verified from the available evidence.

02 / PROOFS

Proofs

Contains unverified proofs

The theoretical proof of the effective bound and the Galois obstruction is correct. The computational proof of the quintic classification cannot be independently checked from the supplied data: its LLL bounds, billion-case exclusion, and Magma exhaustions lack reproducible certificates. Three unique notation corrections are harmless.

Lemmas 5–6 and Propositions 1–2: Unit matching and logarithmic lower bounds prove Theorem 1

Pages 6–12 · Section 3 · arXiv:2608.15903v1

Correct and complete

Regulator cofactors give both directions of the exponent-to-height comparison. In each ordering of the four conjugate terms, the quotient closest to one yields a nonzero logarithmic form; the selected algebraic numbers and coefficient bound satisfy Matveev's hypotheses, including the root-of-unity term. The resulting lower bound and the elementary upper bound produce the common constants C_4, C_5. Proposition 1's product-formula argument handles every matching pattern, and the final scalar inequality is solved effectively. Every later use depends only on the maximum of these constants over subfields.

Verifiable source: Matveev, explicit lower bound for logarithmic forms

Lemmas 7–8: The block-system and index-two-subfield arguments are complete

Pages 12–13 · Section 4 · arXiv:2608.15903v1

Correct and complete

Applying every normal-closure automorphism to $n = \varepsilon^{(1)} + \varepsilon^{(i)}$ partitions all embeddings into two-element blocks, so the permutation group embeds in the stated wreath product. The same symmetry gives $f(x) = f(n - x)$; after shifting by $n/2$, the irreducible polynomial is even, and its degree-$d/2$ polynomial in the square is irreducible. This produces the proper index-two subfield and proves both advertised consequences.

Lemma 9 · LLL reductions: The numerical reduction to the bound 111.49 is not reproducible from the paper

Pages 13–16 · logarithmic matrix, Equations (9)–(12), Table 2, and Lemma 9 · arXiv:2608.15903v1

Not able to verify

The paper prints only a 30-bit logarithmic matrix but says that the decisive lattice reductions used 1000-bit and 10000-bit arithmetic. Table 2 records the scaling choices and output bounds, not the LLL-reduced bases, Gram–Schmidt bounds, or rigorous intervals certifying the nonzero logarithmic forms. Those data are necessary to verify each application of Lemma 4 and the final bound $B = 111.49$. The method is mathematically applicable, but the reported numerical conclusion cannot be audited from the manuscript alone.

Lemma 10 and final unit-equation search: The two exhaustive computations have no checkable certificate

Pages 16–17 · Equations (13)–(15), Lemma 10, and proof of Theorem 2 · arXiv:2608.15903v1

Not able to verify

Lemma 10 rests on enumerating 1,676,902,296 lattice points and finding none that passes inequality (13); the paper reports a 23-day run but provides no enumeration code, exact interval tests, or digest of the checked set. The final count then relies on Magma's 'UnitEquation' output for every $1 \leq n \leq 1000$, again without scripts or machine-checkable output. Consequently neither the exclusion of $|n| > 1000$ nor the completeness and equivalence reduction of the 255 listed pairs can be independently verified.

Lemma 6, Section 5, and Lemma 8: Three symbols have uniquely determined local corrections

Pages 10, 13, and 14 · last case of Lemma 6, proof of Lemma 8, and computation of C_4 · arXiv:2608.15903v1

Typo

The last case of Lemma 6 prints 'provided that $X > C_5 = ert(C_3 + 1)X$'; the trailing X must be deleted, giving $C_5 = ert(C_3 + 1)$. In the numerical specialization, 'provided that $X < C_5 = 68.133$' must read $X > C_5$, as required by the same case split. Finally, after shifting a minimal polynomial by $n/2$, Lemma 8 should state $h \in \mathbb{Q}[x]$, not necessarily $h \in \mathbb{Z}[x]$. The irreducibility and degree argument uses only rational coefficients, so these corrections do not change any conclusion.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2608.15903v1
2. Matveev, explicit lower bound for logarithmic forms

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

$$\ldots$$

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.