MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Mixing for Free Semigroup Actions of Blaschke Products on the Circle

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Paper	arXiv:2608.13876v1
Source version	Submitted 13 August, 2026; announced August 2026.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	August 14, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The classification of free-semigroup mixing rates by the multipliers at the common interior fixed point, the sharp double-exponential exponent, the stated rigidity and non-stability consequences, and the spectral conclusions are supported. Two local notation defects in the correlation convention and an impossible displayed range for an auxiliary radius have evident corrections and do not alter any mathematical conclusion.

Theorem 1.4 and Corollary 1.5: Classification by the multipliers at the common interior fixed point

Pages 5 and 11–17, 26–28 · Theorem 1.4, Sections 3–4 and 8 · arXiv:2608.13876v1

Correct

After simultaneous Möbius normalization, every generator fixes zero and preserves Lebesgue measure. A generator with multiplier of modulus one is a rotation and prevents mixing. If every multiplier has modulus below one, the Schwarz-lemma estimate gives a uniform exponential operator-norm bound for every word. If every multiplier is zero, composition raises the order of vanishing by at least the minimal local degree p at each letter, yielding

$$\|L^a\| \leq C \exp(-\beta p^{|a|}).$$

Conversely, repeating a generator with nonzero multiplier produces a matching exponential lower bound and rules out double-exponential mixing. Section 8 transfers all three alternatives to the harmonic measure at the original common fixed point, with the required analytic-norm comparison.

Theorem 5.1 and Corollary 5.2: Sharpness of the exponent and rigidity at maximal local degree

Pages 18–20 · Section 5 · arXiv:2608.13876v1

Correct

For a map with local expansion $\phi(z) = c_p z^p + O(z^{p+1})$, iteration has leading term

$$c_p^{(p^n-1)/(p-1)} z^{p^n}.$$

The lacunary test observable isolates that coefficient along an infinite subsequence, while the remaining Fourier tail has a strictly smaller double-exponential base. This proves that no exponent larger than $\log p$ is possible. Since the local degree satisfies $p \leq q$ for a degree-q Blaschke product, attaining $\log q$ forces every zero to lie at the fixed point and hence gives the stated Möbius-conjugate affine model.

Propositions 6.1 and 6.2: Failure of stability and nowhere density

Pages 20–22 · Section 6 · arXiv:2608.13876v1

Correct

The two explicit degree-three families converge to z^3 in the C^1 topology, while their local degrees at zero are respectively two and one, so the multiplier criterion places them on opposite sides of double-exponential mixing. Their local degrees also exclude Möbius conjugacy to z^3; the cited Shub–Sullivan rigidity result upgrades any absolutely continuous conjugacy between the expanding Blaschke products to precisely such a conjugacy. Finally, within the normalized Blaschke products the double-exponential locus is the closed condition $\phi'(0) = 0$, and an arbitrarily small displacement of all but one zero at the origin leaves the normalized class while making the derivative nonzero. This proves nowhere density.

Verifiable source: Shub and Sullivan, Expanding endomorphisms of the circle revisited

Theorems 7.1 and 7.2: Spectrum, joint spectral radius, and the quasi-nilpotent algebra

Pages 22–25 · Section 7 · arXiv:2608.13876v1

Correct

On the positive and negative Fourier subspaces the compact precomposition operator is triangular with diagonal entries λ^m and $\overline{\lambda}^m$, respectively, which gives the claimed spectrum together with zero. For a word, the multiplier is the product of the letter multipliers. The Berger–Wang formula therefore identifies the joint spectral radius with $\max_j |\lambda_j|$. When every multiplier vanishes, the word-norm estimate is uniform and double exponential in word length; expanding a power of any zero-constant noncommutative polynomial into words then forces its spectral radius to be zero.

02 / PROOFS

Proofs

Correct

The proofs of the classification, sharp exponent, stability consequences, and spectral results are complete. The manuscript has a consistent but local mismatch between its printed correlation convention and the Hermitian pairing used in the calculations, and one auxiliary-radius interval is mistyped. Both are uniquely and harmlessly repairable; neither requires a change to any theorem.

Sections 3–5: Operator bounds, classification, and sharpness

Pages 11–20 · Sections 3–5 · arXiv:2608.13876v1

Correct and complete

The Hilbert–Schmidt estimate is reduced to Fourier coefficients of the iterated Blaschke products. In the zero-multiplier case, the order-of-vanishing cutoff makes the remaining geometric sums double exponentially small. In the strict-contraction case, moving the contour to a radius between θ and one gives an exponentially small factor per letter. Lemma 3.2 supplies the necessary lower bound when a multiplier is nonzero, and the lacunary construction in Theorem 5.1 prevents cancellation from obscuring the leading coefficient. With the two local notation corrections recorded below, every required implication and quantitative comparison closes.

Sections 7–8: Spectral and conjugation arguments

Pages 22–28 · Sections 7–8 · arXiv:2608.13876v1

Correct and complete

Compact triangularity determines the one-generator spectrum, and the product formula for derivatives plus Berger–Wang gives the joint spectral radius. The proof for zero-constant operator polynomials correctly controls every term in the nth power by words of length at least n. For a common fixed point away from zero, the single Möbius conjugation transports the action and harmonic measure simultaneously. Lemma 8.1 proves the only nonautomatic point: precomposition sends H_θ boundedly into an $H_{\theta'}$ corresponding to a slightly smaller annulus.

Correlation convention: The printed pairing omits the conjugation used throughout the calculations

Pages 4, 13, 18, and 26 · definition of $C_{f,g}$ and later uses · arXiv:2608.13876v1

Typo

The definition prints an absolute value around

$$\int (f \circ \tau(a)) g \, d\nu_w - \int f \, d\nu_w \int g \, d\nu_w,$$

but the proof then identifies the first integral with the Hermitian L^2 inner product, Lemma 4.1 extracts coefficients using z^{-j}, and Theorem 5.1 pairs matching positive Fourier modes. These calculations use the convention

$$C_{f,g}(a) = \left| \int (f \circ \tau(a)) \overline{g} \, d\nu_w - \left(\int f \, d\nu_w \right) \overline{\left(\int g \, d\nu_w \right)} \right|.$$

Inserting the two conjugations makes every displayed computation consistent. Equivalently, one may retain the printed bilinear convention and replace each later g by $\overline{g}$; the function classes and norms are conjugation-invariant. This is a notation-level correction only and leaves all estimates and statements unchanged.

Proof of Theorem 3.1, Part 2: The initial range printed for the contour radius is empty

Page 17 · opening sentence of the proof in Section 4.2 · arXiv:2608.13876v1

Minor formal correction

The sentence begins with "for any $0 < \theta < 1$ and any $1 < r < 1$." Equation (4.2) immediately below chooses the radius in the nonempty interval $r \in (\theta, 1)$, and every preceding and subsequent contour estimate uses exactly that range. Replacing $1 < r < 1$ by $\theta < r < 1$ is the unique local correction and changes no argument.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2608.13876v1
2. Shub and Sullivan, Expanding endomorphisms of the circle revisited

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.