MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Mixing Properties of Random Laguerre Tessellations

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Prompt	Standardized prompt
Novelty cutoff	August 13, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains wrong statements

The moment condition does correctly make the stationary Laguerre generator admissible, and the subsequent localization scheme gives substantial support for preservation of ergodicity, mixing, and α-mixing. Theorem 4 is nevertheless false on its printed domain: for a locally finite weighted configuration the displayed family of positive-dimensional cells need not be closed in the Fell hyperspace, so $L(\varphi)$ need not even belong to the asserted codomain. Because all three mixing implications invoke that theorem, their proofs require a measurable-map result restricted to regular or tempered configurations before those implications can be fully verified.

Theorem 1(a), admissibility clause: The moment condition yields a well-defined Laguerre tessellation

Pages 4–6 and 10 · Theorem 1(a), Lemmas 2 and 5 · arXiv:2608.13235v1

Correct

Applying the tempered-configuration estimate to the transformed marks $\sqrt{M_-}$ gives eventual control $m \geq -(\|x\| - k)^2$ for distant generators. This makes every sublevel set of the power distance locally finite. Stationarity and nondegeneracy give $\operatorname{conv}(\eta') = \mathbb{R}^d$ almost surely, and the spatial ergodic theorem places the transformed configuration in $\mathcal{M}_{\text{temp},\delta}$ under $\mathbb{E}M_-^{(d+\delta)/2} < \infty$. Proposition 3 then supplies compact, locally finite Laguerre cells covering space.

Theorem 4: The Laguerre map does not take every simple marked configuration into the claimed hyperspace

Page 6 · Theorem 4 · arXiv:2608.13235v1

Incorrect

Let $d \geq 2$, $e_1 = (1, 0, \ldots, 0)$, $x_n = ne_1$, $b_1 = 0$, $b_n = -2\sum_{j=1}^{n-1} j^{-1}$, and $m_n = b_n - n^2$. The configuration $\varphi = \{(x_n, m_n) : n \geq 1\}$ has a simple locally finite ground set and hence lies in the theorem's domain. After subtracting the common term $\|z\|^2$, its power functions are the affine functions $-2nz_1 + b_n$. Consecutive functions cross at $z_1 = -1/n$, so the positive-interior Laguerre cells are

$$(-\infty, -1] \times \mathbb{R}^{d-1} \quad \text{and} \quad [-1/(n-1), -1/n] \times \mathbb{R}^{d-1} \quad (n \geq 2).$$

The latter cells converge in the Fell topology to $\{0\} \times \mathbb{R}^{d-1}$, which is not one of the cells. Thus $L(\varphi)$ is not closed in $\mathcal{F}'_d$ and is not an element of $\mathcal{F}(\mathcal{F}'_d)$, contrary to Theorem 4. A repair must restrict the domain, for example to configurations whose diagrams are locally finite tessellations, and prove measurability on that restricted measurable set; merely changing the proof's two displayed symbols does not repair the printed theorem.

Verifiable source: Reviewed manuscript, version 1

Theorem 1(a)–(c), mixing clauses: The factor and localization conclusions need a valid restricted measurability theorem

Pages 4 and 10–15 · Theorem 1 and its proof · arXiv:2608.13235v1

Not able to verify

Ergodicity and ordinary mixing would follow immediately if L were a measurable translation-equivariant map on an almost-sure set containing the tempered configurations. The α-mixing proof similarly localizes inner and outer cell events and then applies measurability to truncated diagrams. Theorem 4 does not provide the required premise because it is false on its stated domain, and the paper does not define and prove measurability of a repaired restriction. The localization estimates themselves do not furnish that missing measurable-factor step. No counterexample to the three implications was found, but the supplied argument does not formally establish them until this gap is repaired.

Lemma 17: The Poisson moment threshold is exact

Pages 15–16 · Lemma 17 · arXiv:2608.13235v1

Correct

For a marked Poisson process, $\mathbb{E}M_-^{d/2} < \infty$ makes the number of generators whose power ball reaches a fixed compact set finite, verifying (R1), while stationarity verifies (R2). If that moment diverges, the Poisson void formula gives $\inf_{x\in\eta} \rho(0,x) = -\infty$ almost surely; hence no cell contains the origin and the diagram cannot be space filling. This proves both directions of the stated threshold.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The temperedness and void-probability arguments are correct, and the geometric localization lemmas support the intended α-mixing reduction. The proof of Theorem 4 fails at the level of its codomain and also contains two reversed extrema and a sentinel collision. Consequently the measurable-factor steps in all three mixing proofs are incomplete. The reversed extrema are reported as typos because each has a uniquely determined local correction, but those corrections alone do not fix the theorem.

Proof of Theorem 4: The proof assumes the cell family is closed and uses possible cells as exceptional sentinels

Pages 7–10 · Lemmas 7–8 and proof of Theorem 4 · arXiv:2608.13235v1

Incorrect as written

The construction treats $L(\varphi)$ as an element of $\mathcal{F}(\mathcal{F}'_d)$ before proving that its set of cells is closed; the explicit configuration in the statements finding shows that this can fail. In addition, Λ replaces empty-interior cells by $B(0,2)$ and $\widetilde{\Lambda}$ replaces non-atoms by $B(0,1)$, after which κ deletes both values. A genuine Laguerre cell can equal either ball: countably many locally finite distant generators can provide a dense family of tangent half-spaces. Thus the construction can delete a real cell. Nonconvex closed sentinels would repair that local collision, but a domain restriction and a proof that the restricted cell family is hyperspace-valued are still required. Repair classification: no complete repair is supplied.

Lemma 7: A supremum is printed where an infimum is required

Page 7 · proof of Lemma 7 · arXiv:2608.13235v1

Typo

For $g(z) = \rho(z,x) - \min_i \rho(z, y_i)$ on a compact C, the condition $g(z) > 0$ for every $z \in C$ is equivalent to $\inf_{z\in C} g(z) > 0$, not to $\sup_{z\in C} g(z) > 0$ as printed. Replacing 'sup' by 'inf' gives the intended measurable continuous minimum and repairs this step mechanically.

Proof of Theorem 4: The finite partial union uses the wrong extremum

Pages 9–10 · definition of L_1^n · arXiv:2608.13235v1

Typo

The upper index of the purported finite partial union is printed as $\max\{n, \varphi(\mathbb{R}^d \times \mathbb{R})\}$. It must be $\min\{n, \varphi(\mathbb{R}^d \times \mathbb{R})\}$: with the printed maximum it is already infinite whenever the configuration has infinitely many atoms, so the claimed measurability of the finite union is circular. The following identity $L = \bigcup_n L_1^n$ determines the correction uniquely.

Proof of Theorem 1(a) and (b): The ergodic and mixing factor argument lacks its measurable map

Page 10 · proof of Theorem 1(a) and (b) · arXiv:2608.13235v1

Incomplete as written

After correctly proving almost-sure temperedness, the proof substitutes $L^{-1}(A)$ and $L^{-1}(B)$ into the definitions of ergodicity and mixing. That substitution requires the restricted Laguerre map to be measurable. The only cited justification is Theorem 4, which is false on its printed domain. A valid repair is plausible—prove measurability on the measurable set of regular or tempered configurations and extend the map arbitrarily off that set—but neither the measurable domain nor the extension argument is supplied.

Proof of Theorem 1(c): The localization estimate is conditional on the same missing measurability repair

Pages 10–15 · Lemmas 9–16 and proof of Theorem 1(c) · arXiv:2608.13235v1

Incomplete as written

Lemmas 10–14 correctly turn an abnormally long or displaced cell into a growing vacant region, whose probability tends to zero by stationarity. Lemmas 15–16 then identify the inner and outer diagrams on the high-probability localization events, yielding the displayed bound by α_η. The probabilities and preimages in this chain are asserted measurable through Theorem 4. Since that theorem fails and no restricted replacement is proved, the final supremum argument has an unresolved formal prerequisite. The geometric and probabilistic estimates otherwise match the stated definition $\alpha_\eta(c, \infty; \Delta)$.

Lemma 18: The separation radius is described by an insufficient condition

Page 16 · proof of Lemma 18 · arXiv:2608.13235v1

Minor formal correction

The proof chooses a_0 only so that $\mathrm{dist}(B_1, B_2) \leq a_0$, then uses independence of the two marking families for $z \notin B(0, a_0)$. That inequality does not ensure $(B_1 + z) \cap B_2 = \varnothing$. Choose instead $a_0 > \sup\{\|y - x\| : x \in B_1,\ y \in B_2\}$, which is finite because both sets are bounded. The omitted integral over the fixed ball then vanishes after normalization, and the rest of the argument is unchanged.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Reviewed manuscript, version 1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.