MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Edit distance exponents for irrational rotations

Andrew Best, Yuval Peres

Paper	arXiv:2608.11763v1
Source version	Submitted 12 August, 2026; announced August 2026.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	August 12, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The four one-dimensional edit-distance exponents, their all-β golden-mean regime, the sharp exceptional-parameter bounds, the circle-map and Sturmian consequences, and the almost-everywhere d-torus exponent are correct. The continued-fraction, matching, probabilistic, and discrepancy estimates give the stated exponents with the required quantifiers. One proof of an auxiliary upper bound omits the regime in which a convergent denominator exceeds the averaging length, but the omitted case has a verified stronger estimate and does not change Proposition 5 or any other statement.

Theorem 1 and Corollary 2: Typical coding parameters and the multifractal dimension formula

Pages 2 and 8–23 · Theorem 1, Corollary 2, and Sections 3–7 · arXiv:2608.11763v1

Correct

The approach-and-follow estimate gives the universal upper bounds

$$\overline{\Gamma}(\alpha,\beta) \le \frac{\mu(\alpha)-1}{\mu(\alpha)}, \qquad \underline{\Gamma}(\alpha,\beta) \le \frac{1}{2}.$$

The common-subsequence lemma converts small edit distance into concordance of most short blocks. Away from the summable exceptional sets for $\operatorname{dist}(\beta, \Lambda(q_m))$ and $\|q_m\beta\|$, Lemmas 6.2 and 7.4 give the matching diameter and typical-pair lower bounds. Weyl equidistribution of $(q_m\beta)$ on a subsequence realizing $\mu(\alpha)$ supplies the typical-pair limsup lower bound. The positive-measure subsequential conclusions become almost-everywhere conclusions because the exponent functions are invariant under the ergodic independent-shift $\mathbb{Z}^2$-action. Finally, $\gamma = (\mu-1)/\mu$ is equivalent to $\mu = 1/(1-\gamma)$, so Güting's exact-exponent dimension $2/\mu$ gives $2-2\gamma$, including the Liouville endpoint by its known zero dimension.

Verifiable source: Full paper, version 1

Theorems 3–4 and Proposition 5: Uniform lower bounds, the golden-mean threshold, and exceptional parameters

Pages 3 and 24–36 · Theorems 3–4, Proposition 5, and Sections 8–9 · arXiv:2608.11763v1

Correct

The signed clean-block argument in Lemma 8.2 and its probabilistic forms control every coding length β, with no metric exclusion. Optimizing the convergent scale gives the threshold $\rho^2-\rho-1=0$, hence $\rho=\varphi$. For $\rho>\varphi$, the round-and-synchronize estimate applies whenever β is eventually $O(q_{m+1}^{-1})$-close to the length-q_m orbit. The nested construction in Proposition 9.3 provides a perfect set of such parameters, while prescribed partial quotients give uncountably many α with steady growth $q_{m+1}=q_m^{\rho+o(1)}$. The optimizations then yield exactly

$$\frac{\rho^2}{2\rho^2-1} \quad \text{and} \quad \frac{\rho}{\rho^2+\rho-1}.$$

Proposition 5 follows from the same synchronization lemma; the missing case in its printed proof is repaired in Part 2.

Verifiable source: Full paper, version 1

Corollary 6: Transfer to aperiodic circle homeomorphisms

Pages 3 and 36–37 · Corollary 6 and its proof · arXiv:2608.11763v1

Correct

For a dense-orbit aperiodic circle homeomorphism, choose the orientation-preserving conjugacy h to the rotation $R_{\omega(f)}$ with $h(0) = 0$. Then

$$\mathbf{1}_{[0,\eta)}(f^n x) = \mathbf{1}_{[0,h(\eta))}(R^n_{\omega(f)} h(x)),$$

so the word sets agree exactly and Theorem 1 applies with $\beta = h(\eta)$. The unique invariant measure satisfies $\nu(h^{-1}E) = \mathcal{L}(E)$, giving the stated ν-almost-everywhere parameter result. If $r > \mu(\omega(f))$, choose a Diophantine exponent strictly between $\mu(\omega(f))$ and r; the cited smooth-conjugacy theorem gives a C^1 conjugacy, which transfers null sets to Lebesgue-null sets as claimed.

Verifiable source: Katznelson–Ornstein, smoothness of the circle conjugacy

Theorem 7: The common exponent for typical rotations of $\mathbb{T}^d$

Pages 3–4 and 38–42 · Theorem 7 and Theorems 11.7, 11.10 · arXiv:2608.11763v1

Correct

For a rotation vector with discrepancy excess $\Delta_q = q^{o(1)}$, the covering consequence of discrepancy finds an orbit shift within distance $O((\Delta_q/q)^{1/d})$. Counting visits to the $O(\epsilon^{1-d})$ boundary cubes gives the upper exponent $d/(d+1)$. Conversely, equal length-k box codings force the two points within $O(\Delta_k/k)$; Lemma 4.3 and a union bound therefore give a summable small-edit probability at the optimized scale $k = N^{1/(d+1)}$. Borel–Cantelli yields the typical-pair lower exponent $d/(d+1)$, and the ordering between typical and diameter exponents forces all four exponents to coincide. Beck's metric discrepancy theorem supplies $\Delta_q = q^{o(1)}$ for almost every rotation vector.

Verifiable source: Beck, metric discrepancy of Kronecker sequences

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The central proof chains for the one-dimensional typical results, the golden-mean transition, the sharp exceptional constructions, circle conjugacy, and multidimensional discrepancy bounds are correct and complete. The proof of Proposition 9.9, used for Proposition 5(a), incorrectly asserts that an adapted exponent θ is at most one and therefore omits the possible large-denominator regime. That regime is repaired directly from Lemma 9.1 and in fact gives a stronger estimate.

Core one-dimensional proof chain: Approach-and-follow, concordance, and clean-block estimates close

Sections 3–8 · arXiv:2608.11763v1

Correct and complete

Lemma 3.2 balances the time needed to approach a second orbit against visits to the two coding boundaries. Lemma 4.3 correctly extracts at least $N - 3k\ell$ concordant starting positions from a common subsequence of defect at most ℓ. Lemma 6.2 uses the separation of β from $\Lambda(q_m)$ to produce a discordant set of measure greater than $3N^{-\epsilon}$, and Lemma 7.4 obtains the complementary typical-pair estimate by comparing the number of ones in q_m-blocks. In Section 8, opposite signs of clean blocks force matching-shift variation, while missing a sign forces many non-clean blocks; Lemma 7.2 controls both by the edit defect. All exponent inequalities, summability conditions, endpoint conventions for $\mu(\alpha) = \infty$, and uses of the almost-everywhere invariant exponents are consistent.

Round-and-synchronize lemma: The periodic rounding and drift corrections give the stated edit bound

Pages 31–32 · Lemma 9.1 · arXiv:2608.11763v1

Correct and complete

Rounding β to j_0/q_m changes at most $4b$ symbols in each block of length q_{m+1}. Rounding α to p_m/q_m produces a q_m-periodic word. Each accumulated displacement of $1/q_m$ is synchronized by deleting q_{m-1} symbols, using $p_{m-1}q_m - p_m q_{m-1} = (-1)^m$, and consecutive corrections are at least q_{m+1} positions apart. The common-subsequence construction therefore proves

$$\operatorname{diam}_E(\mathcal{W}_N) \leq q_m + C(q_{m-1} + b)\left\lceil \frac{N}{q_{m+1}} \right\rceil$$

for both signs of $\alpha - p_m/q_m$.

Proof of Proposition 9.9: The adapted convergent can be larger than the averaging length

Page 36 · proof of Proposition 9.9, immediately after choosing $q_m < N^{1/3} \leq q_{m+1}$ · arXiv:2608.11763v1

Incorrect as written · verified repair

The proof writes $q_{m+1} = N^\theta$ and asserts $\theta \in [1/3, 1]$. There is no upper bound $q_{m+1} \leq N$; for a large partial quotient one can have $q_m < N^{1/3}$ but $q_{m+1} > N$, so $\theta > 1$. The subsequent estimate $q_m N^{1-\theta}$ also cannot absorb the leading q_m term in that regime. The repair is verified and stronger: if $q_{m+1} > N$, Lemma 9.1 with $b = 2$ gives

$$\operatorname{diam}_E(\mathcal{W}_N) \leq q_m + C(q_{m-1} + 2) \left\lceil \frac{N}{q_{m+1}} \right\rceil = q_m + C(q_{m-1} + 2) \ll q_m < N^{1/3}.$$

If $q_{m+1} \leq N$, then $\theta \in [1/3, 1]$ and the paper's two-case optimization applies unchanged, giving $O(N^{2/3})$. Thus Proposition 9.9 and Proposition 5(a) are correct, but the printed proof omits a necessary case.

Multidimensional proof chain: Discrepancy and transference inputs are applied with matching scales

Sections 11.2–11.4 · arXiv:2608.11763v1

Correct and complete

The radius in Lemma 11.3 is chosen so that a slightly enlarged cube has q times its volume greater than Δ_q, and passage to the closed cube is valid. Lemma 11.5 counts disagreements in the boundary neighborhood with the correct factor ϵ^{1-d}. For the lower bound, the box in Lemma 11.8 has volume $\Delta_k/(k\beta_i)$, so discrepancy forces a coding mismatch whenever the points are farther than $C_3\Delta_k/k$. The optimized Borel–Cantelli calculation then yields $d(1-\delta)/(1+d(1-\delta))$. In the non-singular extension, the cited homogeneous-to-inhomogeneous transference gives the required covering radius, while open cubes of radius $\psi_\alpha(q)/2$ contain at most one point of a length-q orbit segment; the resulting subsequential liminf and typical limsup bounds follow.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1
2. Katznelson–Ornstein, smoothness of the circle conjugacy
3. Beck, metric discrepancy of Kronecker sequences

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.