MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Visible Measures along $\Omega(n)$ and Distribution of Horocycle Orbits

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Paper	arXiv:2608.11382v1
Source version	Submitted 11 August, 2026; announced August 2026.
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Prompt	Standardized prompt
Novelty cutoff	August 11, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The ergodic-visible-measure theorem, the symbolic counterexample, the classification and realization results for horocycle Ω-limits, the exclusion result for bounded geodesic orbits, and the pointwise-divergence corollary are correct. Several printed proofs require substantive repairs, recorded in Part 2, but the repairs can be verified without changing any central conclusion. The use of cusp Dirac masses also requires the explicitly intended compactified ambient space.

Theorem 1.1: Every ergodic standard visible measure remains visible along $\Omega(n)$

Page 2 · Theorem 1.1; pages 7–10 · proof · arXiv:2608.11382v1

Correct

The theorem remains valid for the stated σ-compact metric spaces, although the printed $C_c(X)$ argument is not valid at that generality. A verified repair uses that a σ-compact metric space is separable, fixes a countable weak-convergence-determining family of bounded continuous functions, and applies the same Egorov and Gaussian-block construction to its first m members. Quasi-genericity gives the required lower bound for the open good set directly by Portmanteau. To obtain tightness, at stage m include continuous cutoffs that equal one on compact sets carrying almost all of μ and vanish outside shrinking neighborhoods of those sets. Each Ω-empirical measure has finite support; the union of its high-mass support portions in neighborhoods of radius tending to zero has compact closure. The resulting diagonal sequence is tight and converges on the determining family, hence converges weakly to μ.

Verifiable source: Full paper, version 1

Proposition 1.2: Ergodicity cannot be removed

Page 2 · Proposition 1.2; pages 10–11 · symbolic construction · arXiv:2608.11382v1

Correct

The three-symbol block construction is generic for $\frac{1}{3}(\delta_{\overline{0}}+\delta_{\overline{1}}+\delta_{\overline{2}})$: a partial terminal block has length $O(N_{i+1}^{2/3}) = o(N_i)$, so the endpoint calculation extends to every averaging time. The Hardy–Ramanujan window around $\log\log N$ has length $O((\log\log N)^{1/2+1/100})$ and meets at most two symbol blocks. Thus, outside a set of integers of density tending to zero, at least one of the three zero-coordinate cylinder functions has average zero. All three therefore cannot converge to 1/3, and the non-ergodic measure is not in $\mathrm{Acc}_\Omega(x)$.

Verifiable source: Hardy–Ramanujan concentration theorem as used in the paper

Theorems 1.3 and 1.6: Horocycle accumulation measures and the bounded-geodesic exclusion

Pages 2–3 · Theorems 1.3 and 1.6; Sections 5–6 · arXiv:2608.11382v1

Correct

The local Erdős estimate converts Ω-averages into Gaussian-weighted horocycle windows. Quantitative horocycle equidistribution gives the dichotomy between total cusp escape and approximation by a periodic horocycle of period bounded above and below. The matrix factorization in Lemma 5.5 and the second-derivative estimate in Lemma 5.7 then turn the latter case into exactly

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-r^2/2}\nu^i_{s_0-2\log|1+c_0(r-z_0)|}\,dr.$$

Recurrence of the positive geodesic orbit and long-horocycle equidistribution give the asserted nontrivial family and Haar measure. If the full geodesic orbit is bounded, quantitative non-divergence rules out cusp mass and the Diophantine lower bound in Lemma 5.3 keeps the scaled shear parameter away from zero, ruling out a single periodic measure. The formula and constant bookkeeping in the lower-inclusion proof need the verified repairs listed in Part 2, but the repaired chain establishes the printed conclusions.

Verifiable source: Streck, closed-horocycle approximation lemma

Theorem 1.7 and Corollary 1.5: Residual realization and almost-everywhere pointwise divergence

Pages 3 and 25–29 · Theorem 1.7, Propositions 7.1–7.3, and proof of Corollary 1.5 · arXiv:2608.11382v1

Correct

After the repairs in Part 2, the Baire construction realizes every cusp mass and a dense parameter set of the Gaussian mixtures; continuity in the parameters supplies the full upper-bound family. For pointwise divergence, the recurrence compactum and the constant in Theorem 1.3 can be chosen uniformly off a null set. The parameters s_x and c_x then range over a fixed compact subset. A cusp truncation K_R may be chosen so that each mixture assigns at least a constant multiple of e^{-R} to $X \setminus K_R$, while Haar measure assigns at most a constant multiple of e^{-2R} there. A compactly supported continuous cutoff of K_R therefore has different Haar and mixture integrals for every such x, giving two distinct subsequential limits almost everywhere.

Verifiable source: Sarnak, equidistribution of long periodic horocycles

Horocycle notation: The accumulation set must be taken in the cusp compactification

Pages 1–3 · definitions of $\mathrm{Acc}_\Omega(x)$ and Theorems 1.3, 1.6–1.7; page 19 · start of Section 6.1 · arXiv:2608.11382v1

Typo

The general introduction defines $\mathrm{Acc}_\Omega(x)$ using probability measures on X, but Theorems 1.3 and 1.7 include the cusp masses ν^i_∞, which live only on the k-point compactification X_∞. Section 6.1 explicitly says that its limit κ is a measure on X_∞, so the intended correction is unique: in the horocycle results, define $\mathrm{Acc}_\Omega(x)$ as the weak-* accumulation set in $\mathcal{M}(X_\infty)$. This corrects the ambient-space notation and leaves every argument and conclusion unchanged.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The main mechanisms are mathematically sound, but four printed proof segments are not valid as written: the general-space proof of Theorem 1.1 assumes local compactness that was not stated; the uniform constant in Proposition 6.3 is not obtained from the displayed estimate; the residual cusp construction uses an invalid bounded-distance inference and an unnecessarily over-strong auxiliary lemma; and the pointwise-divergence proof reverses a cusp truncation. Each central conclusion nevertheless has a verified repair. Several additional sign and exponent errors are uniquely determined typographical corrections.

Proof of Theorem 1.1: The $C_c(X)$ reduction silently assumes local compactness

Pages 7–9 · first reductions and Equation (11) · arXiv:2608.11382v1

Incorrect as written · verified repair

For a merely σ-compact metric space, it is false that one can choose compact metric balls exhausting almost all mass, and an open set need not admit compactly supported continuous approximants from below. For example, $X = \mathbb{Q}$ with its usual metric is σ-compact, has no compact ball of positive radius, and every nonzero continuous function has noncompact support. Thus the assertions preceding (7) and (11) do not follow from the hypotheses. The repair described in Part 1 is verified: work with bounded continuous determining functions, use Portmanteau for the open good set, and enforce tightness through shrinking neighborhoods of compact high-mass sets and the finite supports of the empirical measures. No local compactness assumption is then needed.

Proof of Equation (2) in Theorem 1.3: The displayed bound does not produce a constant uniform in the target periodic orbit

Pages 22–24 · Proposition 6.3 and the proof of Equation (2) · arXiv:2608.11382v1

Incomplete as written · verified repair

Theorem 1.3 requires one multiplicative constant D for all cusps and all centers s_0. Proposition 6.3 states $D = D_{\Gamma,x}$, but its displayed application of Lemma 5.1 gives an error $C'C^5\bar{r}^{-\alpha}$, where $C = C_p$ depends on the chosen periodic point $p = g_{s_0}(e_i)$. Taken literally, choosing D from this inequality makes it depend on p. In the coordinates used immediately above (22), however, $g_{\log T}v_s(p)$ lies in the fixed ball $B(e, 1)$ and the three widths defining $\widetilde{G}$ are fixed positive constants. Lemma 5.1 therefore supplies a bound with a constant depending only on Γ, not on C_p; the extra factor C^5 can be removed. Choosing D from this uniform estimate verifies Proposition 6.3 with $D = D_{\Gamma,x}$, and the final ratio of the two admissible scaled-shear bounds is then $8D$, as required.

Proof of Proposition 7.1: The claimed bounded perturbation is not bounded when $|rs|$ approaches one

Pages 25–26 · proof of Proposition 7.3 after the Baire construction · arXiv:2608.11382v1

Incorrect as written · verified repair

The proof uses

$$h_r v_s h_{-r} = \begin{pmatrix} 1+rs & -r^2 s \\ s & 1-rs \end{pmatrix}$$

and concludes from $|rs| \le 1$ that $h_{T_i+r}x$ stays a uniformly bounded distance from the shrinking periodic orbit. This inference is false at the displayed endpoint: the diagonal factor in the standard triangular decomposition is unbounded as $1+rs$ or $1-rs$ tends to zero. Apply Proposition 7.3 with the function $w/2$ instead. It still satisfies $t(w(t)/2) \to \infty$, and its conclusion supplies a window of radius $2w(T_i)^{-1}$; restricting to the required radius $w(T_i)^{-1}$ gives $|rs| \le 1/2$. The triangular factors are then uniformly bounded, so the entire required window escapes every compact set.

Lemma 7.4 in the residual cusp construction: Sarnak's theorem does not justify the printed uniform period quantifier

Pages 25–27 · Lemma 7.4 and its use in Proposition 7.3 · arXiv:2608.11382v1

Incomplete as written · verified repair

Lemma 7.4 is stated uniformly for every periodic point of period at least ϵ. The proof produces on the renormalized closed horocycle an interval of length greater than $5k_T$, while that orbit has period asymptotic to $k_T\,\mathrm{per}(p)$. Sarnak's equidistribution of the complete closed-orbit measure does not imply that this particular interval meets B when $\mathrm{per}(p)$ is unbounded. The Baire proof needs only a chosen sequence p_ℓ with periods tending to zero. Choose it with $\mathrm{per}(p_\ell) < 1$ for every ℓ and replace Lemma 7.4 by the corresponding fixed-p statement. Then the interval of length $5k_T$ covers the entire renormalized closed orbit for large T; Sarnak's theorem applies directly, and every density assertion used in Proposition 7.3 follows.

Proof of Corollary 1.5: The final cusp-mass comparison uses the complement of the set that was defined

Pages 28–29 · final paragraph of the proof of Corollary 1.5 · arXiv:2608.11382v1

Incorrect as written · verified repair

The paper defines $B = \{y : d_X(y,e) \le 2C_\epsilon + C_2\}$ but then asserts $\mu_X(B) \asymp e^{-2C_\epsilon}$ and identifies the mixture's B-mass with the Gaussian integral over the small interval around $r = -1/c_x$. A growing metric ball has Haar mass tending to one; both assertions concern its cusp complement. Let K_R be a smooth compact cusp truncation. Standard cusp coordinates give $\mu_X(X \setminus K_R) \ll e^{-2R}$. For

$$I_x = -c_x^{-1} + [-e^{-R}/c_x, e^{-R}/c_x],$$

the Gaussian mass is uniformly $\gg e^{-R}$ because c_x remains in a fixed compact interval, and for $r \in I_x$ the periodic orbit $\nu^i_{s_x - 2\log|1+c_x r|}$ lies beyond K_R once R is large. Hence the mixture has outside mass $\gg e^{-R}$, strictly more than Haar. A continuous compactly supported cutoff equal to one on a slightly smaller truncation then separates the two measures uniformly, proving the corollary.

Proof of Equation (2): Several exponents and signs are transcription errors

Pages 23–24 · Proposition 6.3 and Equations (21)–(23) · arXiv:2608.11382v1

Typo

Four corrections are mechanically forced by the adjacent formulas. First, after applying Lemma 5.2, every displayed $C^{1/2}T_\ell^{-1/2}$ bound in the inclusion in $V_T(p)$ and in the later range for c must read $C^{3/2}T_\ell^{-1/2}$. Second, the definition of G must use $|e^{b/2}-1| < 1/10$, not $|b/2-1| < 1/10$, exactly as in the preceding inclusion. Third, Equation (23) requires $G_{t_\ell}(k) = e^{-k^2/(2t_\ell)}$, not a positive exponent. Fourth, the three Riemann-sum lines that switch $-2\log|1+acu_j|$ to $+2\log|1+acu_j|$ and then to $+\log|1+c'r|$ must retain $-2\log$ throughout. The preceding shearing formula and the final displayed limit already contain these corrected signs, so no estimate or conclusion changes.

Proof of Proposition 7.2: The parameter conversion has two sign mismatches

Pages 25 and 28 · Proposition 7.2 and its proof · arXiv:2608.11382v1

Typo

Proposition 7.2 must display $\nu^i_{s-2\log|1+c(r-z)|}$, with a minus sign, to match Theorem 1.7 and Lemma 5.9. In its proof, put $A = 1-cz$, $s' = s-2\log|A|$, and $c' = c/A$; the paper instead omits the absolute value in s' and inserts one in the denominator of c'. The corrected definitions give the exact identity

$$s' - 2\log|1+c'r| = s - 2\log|1+c(r-z)|$$

for both signs of A. This identity uniquely determines the corrections and completes the already printed Baire argument.

Local proof notation: Three harmless symbols in Sections 5–6 need correction

Pages 14, 16, and 21–22 · Lemmas 5.4, 5.7, and proof of Theorem 1.6 · arXiv:2608.11382v1

Typo

In Lemma 5.4, $|(a+a^{-1})S|$ must be $|(a-a^{-1})S|$, as follows from the polynomial $P(t) = -ct^2 + (a-a^{-1})t$. In Lemma 5.7, the second-derivative bounds must use $|\ell|$ (or apply the estimate to $-f$ when $\ell < 0$). In the last paragraph of the proof of Theorem 1.6, the conclusion must be $|c''| \geq d > 0$, not $c'' \geq d$, because the shear may be negative. Each corrected form is already forced by the preceding line and leaves the argument unchanged.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Hardy–Ramanujan concentration theorem as used in the paper
2. Streck, closed-horocycle approximation lemma
3. Sarnak, equidistribution of long periodic horocycles

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.