MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Absolutely continuous invariant measures and the Collet-Eckmann condition for long-branched Sturmian unimodal maps

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Prompt	Standardized prompt
Novelty cutoff	August 11, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The Sturmian kneading characterization, the arithmetic sufficient conditions for existence and nonexistence of an absolutely continuous invariant probability measure, and the universal failure of the Collet–Eckmann condition are correct. One notation typo in the admissibility proof is reported in Part 2 and does not affect any conclusion.

Theorem 1.1: Characterization by the lower mechanical word

Pages 2 and 9–10 · Theorem 1.1 and its proof · arXiv:2608.10655v1

Correct

When $Q(k) \le 1$, admissibility and $Q(1) = 0$ imply that the binary word $Q(1)Q(2)\cdots$ is lexicographically no larger than each of its shifts. Lemma 2.8 then shows that a Sturmian word with this property is the unique lower mechanical word of its slope, so $Q(k) = \lfloor k\omega \rfloor - \lfloor (k-1)\omega \rfloor$. Conversely, that lower mechanical word is Sturmian and Proposition 3.1 verifies admissibility. Summing the kneading recursion gives $S_k = 1 + k + \lfloor k\omega \rfloor$.

Theorem 1.2: Existence of an absolutely continuous invariant probability measure

Pages 3 and 16–19 · Theorem 1.2 and Section 4 · arXiv:2608.10655v1

Correct

The fixed-order uniform decay-of-geometry estimate supplies $\mu_{m(k)} \le \exp(-b_\ell k)$. Lemma 4.2 controls each central pullback by $\mu_{n+1} \le K_\ell \mu_n^{1/\ell}$, and Lemma 4.1 bounds the cascade length L_k by the appropriate odd continued-fraction coefficient. The hypothesis $\limsup_n \ell^{a_{2n+1}}/n < \eta_\ell$ therefore makes $M_\ell \exp(-b_\ell k/\ell^{L_k})$ smaller than the universal threshold for all large k. Lemma 4.3 propagates this through every cascade, and the Bruin–Shen–van Strien criterion then yields the claimed invariant probability measure.

Verifiable source: Full paper, version 1

Theorem 1.3: Arithmetic sufficient condition for nonexistence of an acip

Pages 3 and 20–22 · Theorem 1.3 and Section 5 · arXiv:2608.10655v1

Correct

The co-cutting formula $R(q_{2n}) = q_{2n+1}$ places the interval from c to $f^{q_{2n}}(c)$ inside the relevant kneading neighborhood. Since consecutive block indices differ by $a_{2n+1}(q_{2n} - p_{2n})$, divergence of the resulting weighted recurrence series forces divergence of the total lengths of the kneading neighborhoods, which is equivalent to nonexistence of an acip. The Nowicki–Przytycki trajectory-recurrence estimate supplies $|f^m(c) - c| \ge C_f \Lambda_f^{-m}$, giving exactly the sufficient series in the theorem.

Theorem 1.4: Failure of the Collet–Eckmann condition

Pages 3 and 24–28 · Theorem 1.4 and Section 6 · arXiv:2608.10655v1

Correct

If a realization were Collet–Eckmann, the interval joining c to $f^m(c)$ would be a monotonicity interval for $f^{R(m)}$. Exponential shrinking and logarithmic recurrence would then bound the Cesaro means of the first-disagreement times $R(m)$. For the long-branched Sturmian kneading sequence, mechanical-word occurrences have lower frequencies bounded by the one-sided closest-return gaps $u_n(\omega)$, and $\sum_n u_n(\omega) = \infty$. Proposition 6.7 consequently forces the same Cesaro means to diverge, which is the required contradiction.

02 / PROOFS

Proofs

Correct

The proofs of all four central theorems and their material combinatorial, geometric, inducing, and recurrence inputs are correct and complete. A subscript in Proposition 3.1 is a uniquely repairable notation typo.

Theorem 1.1 and Theorem 3.8: Kneading characterization and co-cutting formulas

Pages 8–13 · Lemma 2.8, Propositions 3.1–3.2, and Theorem 3.8 · arXiv:2608.10655v1

Correct and complete

Balance and lexicographic minimality force the prefix of length n to contain $\lfloor n\omega \rfloor$ symbols 1, proving the mechanical formula. The cutting-time recursion then telescopes. For co-cutting times, the lower intermediate convergents are precisely the successive one-sided record returns of the rotation; inserting their denominators into the continued-fraction recursions gives $S_{q_{2m+1}-p_{2m+1}} = q_{2m+1}$ and $R(q_{2m}) = q_{2m+1}$.

Proof of Theorem 1.2: Uniform geometry and central-cascade induction

Pages 16–19 · Section 4 · arXiv:2608.10655v1

Correct and complete

The distortion estimate is transported through the nonflat normal form with constants depending only on ℓ, yielding the central-step recursion. The induction in Lemma 4.3 keeps every scaling factor below the required threshold through a cascade. The two cases $a_1 = 2$ and $a_1 > 2$ use respectively a_{2k+1} and the shifted sequence a_{2k-1}; the latter has the same limiting upper bound after reindexing. Thus the stated arithmetic hypothesis covers every sufficiently deep level.

Proof of Theorem 1.3: Inducing summability and quantitative recurrence

Pages 20–22 · Section 5 · arXiv:2608.10655v1

Correct and complete

Summation by parts proves that finiteness of the long-branched inducing lift is equivalent to $\sum_k |J_k| < \infty$. The nested-neighborhood estimate and the exact co-cutting return times give the lower series in Proposition 5.4. Nonflatness supplies the global derivative bound required for the cited trajectory-recurrence theorem, so Lemma 5.6 converts this lower series into the one printed in Theorem 1.3.

Proof of Theorem 1.4: Contradictory mean first-disagreement bounds

Pages 24–28 · Section 6 · arXiv:2608.10655v1

Correct and complete

Lemma 6.1 verifies monotonicity up to the first itinerary disagreement. The cited Collet–Eckmann shrinking and recurrence estimates then give a uniform upper bound for $N^{-1} \sum_{m \le N} R(m)$. On the combinatorial side, unique ergodicity gives the stated lower occurrence frequencies, the record-return argument proves $\sum_n u_n(\omega) = \infty$, and the distinct shifts $m_j < 2j$ transfer those occurrences to a divergent lower bound for the same Cesaro means. The contradiction covers every irrational angle and finite critical order.

Proposition 3.1: The displayed order relation has the wrong subscript

Page 10 · final display in the proof of Proposition 3.1 · arXiv:2608.10655v1

Typo

Printed: $\big(Q_\gamma(m+j)\big)_{j\geq 1} \succeq_{\mathrm{pl}} \big(Q_\gamma(j)\big)_{j\geq 1}$. Correction: replace the subscript pl by lex. The paragraph proves a lexicographic comparison, Theorem 2.1 is stated with $\succeq_{\mathrm{lex}}$, and the proof of Theorem 1.1 uses the corrected notation. The intended correction is unique and changes no inference.

Verifiable source: Full paper, version 1

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ *is proved for every sufficiently small delta* > 0 *and* P *is immediately monotone in the required direction, then the claim that* $P(delta)$ *holds for every delta* > 0 *requires no correction; the omitted monotonicity step is exposition rather than an audit finding.*

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ *and subsequently applied at* $t = 0$ *remains a formal scope mismatch unless the endpoint case was separately established before the application.*

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.