MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Entropies of compact subsets and supported measures

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Paper	arXiv:2608.09702v1
Source version	Submitted 10 August, 2026; announced August 2026.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	August 10, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

For nonempty compact K, the upper-capacity and packing entropy equivalences, the Bowen-entropy amplification, and the compact counterexample to the Bowen converse are correct. Theorem 1.3 omits compactness from its first displayed hypothesis; this is a local scope correction consistent with the abstract and the proof and does not lower the substantive status.

Theorem 1.1: Upper-capacity entropy has the zero-or-infinite amplification dichotomy

Pages 2 and 6–9 · Theorem 1.1 and Section 3 · arXiv:2608.09702v1

Correct

Weighted atomic embeddings of K^m into $\mathcal{M}(K)$ multiply upper-capacity entropy by at least m, giving infinity whenever K has positive entropy. Conversely, an exponentially large separated family of supported measures is discretized by a Bowen-spanning partition of K and sent into a finite-dimensional ℓ_∞ space. The cited Glasner–Weiss combinatorial lemma forces that partition to have exponentially many atoms, proving positivity of the entropy of K. The contrapositives yield the zero equivalence.

Theorem 1.2: Packing entropy is positive for K exactly when it is positive for $\mathcal{M}(K)$

Page 3 and pages 9–14 · Theorem 1.2 and Section 4 · arXiv:2608.09702v1

Correct

The Dirac embedding preserves pointwise local entropy and gives the forward inequality. For the converse, the compact-set packing variational principle supplies a measure on $\mathcal{M}(K)$ with positive upper local entropy. Its barycenter has zero upper local entropy if K has zero packing entropy; after restricting most of its mass to a compact low-capacity subset, Lemma 4.2 transfers any remaining positive local entropy back to that subset, giving the required contradiction.

Theorem 1.3: Bowen entropy amplification and failure of the converse

Page 3 and pages 14–17 · Theorem 1.3 and Section 5 · arXiv:2608.09702v1

Correct

For compact K, product measures and the Bowen variational principle give $h^B_{\mathrm{top}}(T^{\times m}, K^m) \geq m h^B_{\mathrm{top}}(T, K)$, and the weighted atomic embedding then makes positive entropy on K infinite on $\mathcal{M}(K)$. In the binary-shift example, the two coordinate sets have alternating zero lower densities, so their union has zero Bowen entropy, while a family of mixtures of two Bernoulli product measures embeds full product alphabets of arbitrary finite size in $\mathcal{M}(K)$ and gives infinite Bowen entropy.

Theorem 1.3 hypothesis: Compactness should be retained in the first assertion

Page 3 · statement of Theorem 1.3 · arXiv:2608.09702v1

Minor formal correction

Insert 'compact' in 'If $K \subset X$ is non-empty.' Proposition 5.1 and its proof invoke the compact-set variational principle, and the abstract, the common setup, and the counterexample all concern compact subsets. No claim for arbitrary noncompact sets is established, while the one-word restriction makes the theorem match every use and leaves all advertised conclusions intact.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The upper-capacity and Bowen arguments are correct and complete. The packing-entropy converse has one concrete support defect: a normalized restriction to a merely Borel set need not have support contained in that set. A verified repair is available by taking a compact subset before restricting, but the printed proof is incomplete as written.

Theorem 4.3: Restriction to a Borel set need not produce a measure supported on that set

Pages 11–13 · proof of Theorem 4.3, definition of R_A · arXiv:2608.09702v1

Incomplete as written

The paper defines $\mathcal{M}(A)$ by the condition $\operatorname{supp}\mu \subset A$. Lemma 4.1 produces a Borel set A, but the proof then asserts that $R_A(\nu) = \nu|_A/\nu(A)$ belongs to $\mathcal{M}(A)$. For nonclosed A, the support of this restriction can contain boundary points outside A, so the pushed-forward measure λ need not satisfy the support hypothesis of Lemma 4.2. Repair classification: Verified repair. By inner regularity on the compact metric space, replace A by a compact subset $A' \subset A$ whose barycenter mass remains above the required threshold. Monotonicity preserves $h^{UC}_{\mathrm{top}}(T, A') < c$, $R_{A'}(\nu)$ is supported on A', and all total-variation and local-entropy estimates continue with the same argument after an arbitrarily small adjustment of θ.

Propositions 3.2 and Theorem 3.4: Atomic embeddings and the finite-dimensional separation argument

Pages 6–9 · Section 3 · arXiv:2608.09702v1

Correct and complete

The binary weights make the ordered atomic representation injective even when coordinates coincide, the map is equivariant, and product separated sets give the lower bound. In the converse direction, uniform continuity of finitely many test functions controls the discretization error, the tail of the metric contributes less than $\epsilon/16$, and the Glasner–Weiss lemma applies with target dimension nL, forcing exponential growth of the spanning number of K.

Proposition 5.1 and Theorem 5.3: Product lower bound and binary-shift counterexample

Pages 14–17 · Section 5 · arXiv:2608.09702v1

Correct and complete

For the product measure, the lower pointwise local entropy is superadditive under the max product metric, and integrating plus the compact Bowen variational principle yields the factor m. In the example, each coordinate set has lower density zero along its alternating endpoint subsequence, which supplies arbitrarily long Bowen covers with subexponential weight. The supported mixture map records every coordinate parameter in the first test function, so an n-ball fixes the first n symbols of the product parameter and has mass at most $(r+1)^{-n}$; Lemma 5.2 then gives entropy at least $\log(r+1)$ for every r.

Final invocation in Theorem 5.3: The local-mass bound is Lemma 5.2

Page 17 · final paragraph of the proof of Theorem 5.3 · arXiv:2608.09702v1

Typo

Replace 'Theorem 5.2' by 'Lemma 5.2'. The immediately preceding result with that number is exactly the local Bowen-ball mass criterion applied here, so the correction is unique and harmless.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2608.09702v1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.