MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

The noncommutative topological factor theorem for rank-one product lattices

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Paper	arXiv:2608.07421v1
Source version	Submitted 7 August, 2026; announced August 2026.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	August 7, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The coordinate classification of intermediate reduced crossed products for abstract rank-one product boundaries, its real Lie, tree, and local-field specializations, and the equivalence between the scalar-expectation classification for the full flag action of $\mathrm{SL}_3(\mathbb{Z})$ and ordinary ITAP are correct.

Theorem A: Intermediate crossed products are exactly the coordinate crossed products

Pages 2 and 11–12 · Theorem A and its proof · arXiv:2608.07421v1

Correct

The abstract factor theorem first forces $C^*(E(D))$ to equal one coordinate algebra $C(X_S)$. Suzuki's Fourier reconstruction gives the upper bound $D \subset C(X_S) \rtimes_r \Gamma$. Independently, $D \cap C(X_N) = C(X_T)$ for a coordinate set T, and the upper bound gives $T \subset S$. For every $j \in S$, a coefficient depending on j is sliced at a free point of a different coordinate; the Powers-averaging lemma places that slice in $D \cap C(X_N)$ while it still separates two points differing only at j. Hence $j \in T$, so $S = T$. The diagonal $C(X_S)$ and the canonical group unitaries then generate the reverse inclusion, proving

$$D = C(X_S) \rtimes_r \Gamma, \qquad C^*(E(D)) = D \cap C(X_N) = C(X_S).$$

Verifiable source: Full paper, version 1

Corollary B: Real rank-one product lattices satisfy the abstract hypotheses

Pages 3 and 12–13 · Corollary B and its proof · arXiv:2608.07421v1

Correct

On each real rank-one flag manifold, the Bruhat decomposition gives transitivity on ordered distinct pairs and a regular split-torus element has north–south dynamics. Irreducibility gives dense coordinate projections. Connectedness and center-freeness make every coordinate projection of the lattice injective; real analyticity and faithfulness then make the coordinate actions topologically free. Cowling–Haagerup weak amenability implies (AP), which is preserved by the finite product and inherited by its lattice. Theorem A therefore applies and indexes the whole interval by 2^N.

Verifiable source: Cowling–Haagerup weak amenability theorem

Theorems 5.1 and 5.4: Tree and mixed local-field specializations

Pages 13–16 · Theorems 5.1 and 5.4 · arXiv:2608.07421v1

Correct

For tree boundaries, a hyperbolic automorphism supplies north–south dynamics and the assumed two-transitivity supplies the remaining rank-one boundary condition; compact vertex stabilizers give weak amenability and hence (AP). For adjoint rank-one algebraic groups over local fields, the rank-one Bruhat cell and a split-torus element give the same boundary dynamics. Irreducibility supplies density, Tits simplicity forces injective coordinate projections, the cited fixed-point theorem gives topological freeness, and the archimedean or tree arguments give (AP). Thus both families satisfy every hypothesis of Theorem A.

Theorem C: The scalar-expectation flag classification is equivalent to ordinary ITAP

Pages 4 and 17–21 · Theorem C and Propositions 6.2–6.3 · arXiv:2608.07421v1

Correct

For a topologically amenable compact action with a dense orbit, the orbit representation embeds $C(X) \rtimes_r \Lambda$ into the uniform Roe algebra and identifies elements with scalar Fourier coefficients with its intersection with $L(\Lambda)$. The amenability kernels constructed in Proposition 6.2 show that this intersection is all of $C^*_u(\Lambda) \cap L(\Lambda)$. Consequently, the assertion that every scalar-expectation intermediate algebra is $C^*_\lambda(\Lambda)$ is equivalent to

$$C^*_u(\Lambda) \cap L(\Lambda) = C^*_\lambda(\Lambda),$$

namely ITAP. The full flag action of $\mathrm{SL}_3(\mathbb{Z})$ is minimal and topologically amenable, so this applies. A full parabolic classification would force the scalar case to be the point quotient and hence would imply ITAP.

Verifiable source: Full paper, version 1

02 / PROOFS

Proofs

Correct

The central proofs are correct and complete. Recurrence supplies simultaneous contraction, the resulting closed-equivalence-relation theorem gives the coordinate commutative core, Fourier reconstruction gives the crossed-product upper bound, Powers averaging recovers coordinate slices, and the positive-definite Schur kernels establish the ITAP equivalence.

Proposition 2.1 and Theorems 2.3–2.5: Recurrence yields the coordinate topological factor theorem

Pages 5–8 · Proposition 2.1 and Theorems 2.3–2.5 · arXiv:2608.07421v1

Correct and complete

Poincaré recurrence on G/Γ supplies lattice elements arbitrarily close to the identity on a chosen subproduct and simultaneously contracting prescribed compact sets in all complementary boundaries. Applied twice, this lets any one coordinate of a related pair be varied arbitrarily while all other coordinates coalesce. Therefore every closed invariant equivalence relation forgets exactly a subset of coordinates. Passing to fibers of an equivariant quotient gives the coordinate factor theorem, and Gelfand duality gives the corresponding classification of unital invariant subalgebras of $C(X_N)$.

Propositions 3.2–3.3: Fourier reconstruction supplies the upper bound

Pages 9–10 · Propositions 3.2–3.3 · arXiv:2608.07421v1

Correct and complete

Suzuki's theorem states that, for a group with (AP), an element of a reduced crossed product whose Fourier coefficients all lie in a fixed closed subspace belongs to the closed span of that subspace's Fourier monomials. For an intermediate algebra D, $C^*(E(D))$ is invariant because D contains the implementing unitaries, so the commutative factor theorem makes it $C(X_S)$. Every coefficient $E(du_\gamma^*)$ lies there, and Suzuki's result therefore gives $D \subset C(X_S) \rtimes_r \Gamma$ without a freeness assumption.

Verifiable source: Suzuki, Proposition 3.4

Lemmas 4.1–4.2: Powers averaging recovers coordinate slices inside the intermediate algebra

Pages 10–11 · Lemmas 4.1–4.2 · arXiv:2608.07421v1

Correct and complete

For each nontrivial Fourier mode t, triviality of the stabilizer at the slicing point gives an open set O with $tO \cap O = \varnothing$. Recurrence produces conjugators s_i that are close to the identity in the other coordinates and send $X_k \setminus O$ into pairwise disjoint open sets. The associated group partition satisfies the hypotheses of the support estimate, so the average of every nonzero Fourier term has norm at most $2\|f_t\|/\sqrt{m}$. All but at most one diagonal translate are uniformly close to the desired slice. Letting the polynomial approximation error tend to zero places the exact slice in $D \cap C(X_{N\setminus\{k\}})$.

Lemma 6.1 and Proposition 6.2: The dense-orbit representation saturates the invariant uniform Roe intersection

Pages 17–19 · Lemma 6.1 and Proposition 6.2 · arXiv:2608.07421v1

Correct and complete

The orbit representation is faithful because its diagonal matrix coefficients evaluate the faithful conditional expectation on the dense orbit. Scalar Fourier coefficients are exactly the condition that its matrix entries are constant on right-invariant diagonals, hence that the image lies in $L(\Lambda)$. Topological amenability provides continuous finitely supported probability fields. Their square roots define positive-definite kernels $k_i(r, t)$ of finite diagonal support; the associated unital completely positive Schur multipliers converge in norm on $C^*_u(\Lambda)$. When $T \in L(\Lambda)$, each Schur approximation has coefficients $c_s h_i(s, \cdot)$ and is the image of a finite Fourier polynomial in the crossed product. Closedness then proves

$$\pi_{x_0}(C(X) \rtimes_r \Lambda) \cap L(\Lambda) = C^*_u(\Lambda) \cap L(\Lambda).$$

Proposition 6.3 and proof of Theorem C: Scalar expectation reduces exactly to ITAP

Pages 19–20 · Proposition 6.3 and proof of Theorem C · arXiv:2608.07421v1

Correct and complete

An intermediate algebra has $C^*(E(D)) = \mathbb{C}1$ exactly when all its Fourier coefficients are scalar, equivalently when it is contained in $D_0 = A \cap L(\Lambda)$. Since D_0 itself is intermediate and has scalar expectation, uniqueness of the scalar intermediate algebra is equivalent to $D_0 = C^*_\lambda(\Lambda)$. Proposition 6.2 identifies D_0 with $C^*_u(\Lambda) \cap L(\Lambda)$, so the resulting equality is precisely ordinary ITAP. The flag-action hypotheses cited immediately before Theorem C verify topological amenability and a dense orbit.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1
2. Cowling–Haagerup weak amenability theorem
3. Suzuki, Proposition 3.4

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.