MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Strong averaging principle for multiscale time-inhomogeneous SDEs with multiplicative α-stable noises

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Paper	arXiv:2608.06011v1
Source version	Submitted 6 August, 2026; announced August 2026.
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Prompt	Standardized prompt
Novelty cutoff	August 6, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The two strong averaging theorems and the all-moment clause of the periodic-measure theorem are not verified for every $p \in (1, \alpha)$ under the printed assumptions. Assumption (A4) gives the required dissipation inequality for only one unspecified exponent, while the proof needs it at the exponent selected in each conclusion. The results are verified for each p that separately satisfies $\lambda > q(p)$.

Theorem 2.2: Strong averaging to the time-dependent averaged equation

Pages 6 and 21–25 · Assumption (A4), Theorem 2.2, and its proof · arXiv:2608.06011v1

Not able to verify

The theorem claims the rate

$$\sup_{t\in[s,T]} \mathbb{E}|X_t^\varepsilon - \overline{X}_t^\varepsilon|^p \le C\varepsilon^{(p-1)/(\alpha+p-1)}$$

for every $p \in (1, \alpha)$. Assumption (A4), however, requires $\lambda > q(p)$ only for some $p \in (1, \alpha)$, where $q(p)$ contains $(\alpha - p)^{-1}$ and therefore tends to infinity as $p \uparrow \alpha$. A finite λ can satisfy (A4) at one exponent while failing $\lambda > q(p)$ at another. Lemmas 3.5, 3.7, 3.8, and 3.10 and the block estimate (3.29) use positive exponential contraction $\lambda - q(p) > 0$ at the same exponent as the theorem. Hence the printed assumptions do not establish the full all-p conclusion. The statement is verified after restricting it to each p for which $\lambda > q(p)$; no counterexample to the broader conclusion is supplied.

Verifiable source: Full paper, version 1

Theorem 2.5: Strong averaging to the epsilon-free averaged equation

Pages 7 and 25–28 · Theorem 2.5 and its proof · arXiv:2608.06011v1

Not able to verify

This theorem makes the same assertion for every $p \in (1, \alpha)$ with rate $\varepsilon^{(p-1)/(\alpha+p-1)}$. Its proof uses the p-moment estimates and the averaged coefficients derived from Theorem 3.9, which in turn depend on contraction at that same p. Assumption (A4) supplies this only at an existentially chosen exponent. The Khasminskii argument does establish the displayed conclusion for each p satisfying $\lambda > q(p)$, but the paper does not provide a separate argument covering exponents for which that inequality fails.

Verifiable source: Full paper, version 1

Theorem 3.9: Periodic measure and its moment bounds

Pages 17–20 · Theorem 3.9 · arXiv:2608.06011v1

Not able to verify

The pullback contraction at an exponent admitted by (A4), followed by Jensen's inequality, correctly gives the stated W_1 convergence, existence, uniqueness, periodicity, and continuity. The additional claim that $\int |w|^p \rho_t^x(dw) \le C_{\kappa,p}(1 + |x|^p)$ for every $p \in [1, \alpha)$ is not established under the existential exponent in (A4): Lemma 3.7 derives its uniform p-moment bound using $\lambda > q(p)$. Thus the periodic-measure conclusion is verified, while its full all-p moment clause is not.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The main proof chain is correct for each exponent satisfying $\lambda > q(p)$, but the paper applies that contraction to every $p \in (1, \alpha)$ although (A4) assumes it for only one exponent. The periodic-measure proof also equates unbounded moments after only W_1 convergence; lower semicontinuity gives a verified repair. One domain mismatch and one omitted constant are harmless local corrections.

Use of Assumption (A4): A contraction hypothesis at one exponent is applied at every exponent

Pages 6, 13–17, and 21–28 · (A4), Lemmas 3.5, 3.7–3.10, and proofs of Theorems 2.2 and 2.5 · arXiv:2608.06011v1

Incorrect as written

Writing the dependence explicitly,

$$q(p) = 2^{\alpha-1} C_{\alpha,d_2} S_{d_2} \left((2-\alpha)^{-1} + (\alpha-1)^{-1} + (\alpha-p)^{-1} \right) C_g^{\alpha}.$$

Assumption (A4) states $\lambda > q(p)$ for some $p \in (1, \alpha)$, whereas the cited lemmas and both main proofs substitute an arbitrary selected p and use $e^{-(\lambda - q(p))(t-s)}$ as a decaying factor. Because $q(p) \to \infty$ as $p \uparrow \alpha$, the required sign does not follow for all selected exponents. Downstream dependency: the periodic-measure p-moment estimates, the averaged-drift estimates, (3.29), and both main rates. Repair classification: Verified repair. State (A4) for the exponent used in a conclusion and replace 'for any $p \in (1, \alpha)$' by 'for every $p \in (1, \alpha)$ such that $\lambda > q(p)$.' Every contraction and balancing estimate in the paper then has the needed sign and yields the stated rate.

Proof of Theorem 3.9, Step 4: First-Wasserstein convergence does not imply convergence of higher moments

Page 20 · first display in Step 4 of Theorem 3.9 · arXiv:2608.06011v1

Incorrect as written

The proof writes

$$\int_{\mathbb{R}^m} |w|^p \rho_t^x(dw) = \lim_{n\to\infty} \int_{\mathbb{R}^m} |w|^p P^x(s - n\tau_2, y, t, dw)$$

after establishing only W_1 convergence. For $p > 1$, the integrand $|w|^p$ is unbounded and this equality does not follow from W_1 convergence plus a uniform p-moment bound. Repair classification: Verified repair. Weak convergence and the nonnegative lower-semicontinuous function $|w|^p$ give the Portmanteau inequality with lim inf in the needed direction; combining it with Lemma 3.7 yields the displayed upper bound (3.13). No equality of moments is required.

Verifiable source: Full paper, version 1

Restricted averaging proof chain: Khasminskii discretization and rate optimization

Pages 21–28 · Sections 3.4–3.5 · arXiv:2608.06011v1

Correct and complete

For a fixed exponent with $\lambda > q(p)$, the auxiliary frozen process has the required moment and contraction estimates. The proof controls the discretization, frozen-fast-variable, and averaging-block terms by $\Delta^{(p-1)/\alpha}$ and $\varepsilon\Delta^{-1}$, then chooses $\Delta = \varepsilon^{\alpha/(\alpha+p-1)}$. The analogous periodic averaging estimate for the second system has the same balance. After the exponent restriction stated above, these steps yield both displayed strong rates.

Lemma 3.10: The fast process requires the terminal time to follow the initial time

Pages 14 and 20 · definition of $Y_t^{s,y,x}$ and statement of Lemma 3.10 · arXiv:2608.06011v1

Minor formal correction

The lemma quantifies over arbitrary $s, t_1, t_2 \in \mathbb{R}$, but $Y_{t_2}^{s,y,x}$ and the two-parameter transition operator were defined only for $t_2 \geq s$. Add the restriction $s \leq t_2$. Every application in the averaging proofs satisfies this order, and the proof then applies without change.

Equation (3.34): The time-Holder estimate omits its coefficient

Page 25 · Equation (3.34) · arXiv:2608.06011v1

Typo

Printed: $|\bar{b}(t,x) - \bar{b}(s,x)| \leq |t-s|^{1/\alpha}$. Assumption (A1) has an arbitrary Holder constant, so the mechanically determined correction is $|\bar{b}(t,x) - \bar{b}(s,x)| \leq C_\kappa |t-s|^{1/\alpha}$. All later estimates already absorb such coefficients into generic constants, so no argument or rate changes.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\mathrm{delta})$ is proved for every sufficiently small $\mathrm{delta} > 0$ and P is immediately monotone in the required direction, then the claim that $P(\mathrm{delta})$ holds for every $\mathrm{delta} > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.