MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Centralizers of sofic approximations of Kazhdan groups

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Prompt	Standardized prompt
Novelty cutoff	August 5, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The finite-level description of the centralizer of a Kazhdan sofic embedding and the deduction that an ergodic centralizer forces the group to be LEF, and residually finite when finitely presented, are correct.

Theorem 3.1: Centralizer rigidity after an essentially equivalent replacement

Pages 6 and 16–17 · Theorem 3.1 and its proof · arXiv:2608.05362v1

Correct

After Kun's decomposition, the finite model is a disjoint union of uniformly expanding components. Clusters of almost equivariant partial bijections form finite groupoids, and their finite full groups act by uniformly approximate permutations. Proposition 4.5 proves both directions needed for centralizer equality: every full-group sequence asymptotically commutes with the sofic image, and every ultraproduct centralizer element is approximated by such a sequence. Uniform flexible stability for finite groups replaces these approximate actions by genuine finite permutation actions after adding $o(|X_n|)$ points. Hence the essentially equivalent embedding satisfies

$$C_{\prod_{\mathcal{U}} \mathrm{Sym}(Y_n)}(\pi'(G)) = \prod_{\mathcal{U}} A_n$$

for finite $A_n \leq \mathrm{Sym}(Y_n)$.

Verifiable source: Full paper, version 1

Theorem A: Ergodic centralizer implies LEF and residual finiteness

Pages 2 and 17 · Theorem A and its proof · arXiv:2608.05362v1

Correct

Theorem 3.1 realizes the centralizer as $\prod_{\mathcal{U}} A_n$. Ergodicity of its Loeb action forces an A_n-orbit occupying asymptotically all of the model. After restricting to those orbits, A_n acts transitively, so its exact permutation centralizer is $N_{A_n}(L_n)/L_n$ acting freely. Proposition 2.7 places G in the corresponding algebraic ultraproduct of finite groups, which is exactly the LEF condition for a finitely generated group. A finite presentation converts sufficiently accurate local embeddings into genuine separating finite quotients, giving residual finiteness.

Verifiable source: Full paper, version 1

Corollary 5.2: Consequence for the Hayes–Kunnawalkam Elayavalli conjecture

Page 17 · Corollary 5.2 · arXiv:2608.05362v1

Correct

The conjectured embedding has exactly the ergodic-centralizer hypothesis of Theorem A, so the LEF conclusion, and the residual-finiteness conclusion in the finitely presented case, follow by direct substitution.

02 / PROOFS

Proofs

Correct

The central proofs are correct and complete. The transitive-centralizer lemma, LEF reduction, expander repair, finite cluster-groupoid construction, exhaustion of the ultraproduct centralizer, and flexible-stability correction all support their stated uses. Two local notation defects are listed below and do not affect the argument.

Propositions 2.7 and 2.9: Transitive finite centralizers and the LEF reduction

Pages 5–6 · Propositions 2.7 and 2.9 · arXiv:2608.05362v1

Correct and complete

For $X_n = A_n/L_n$, uniform almost-commutation with the transitive A_n-action makes the majority value of $g^{-1}\sigma_n(gL_n)$ unique and forces it to be a coset in $N_{A_n}(L_n)/L_n$. Thus the ultraproduct centralizer is the ultraproduct of the exact finite centralizers. Their right actions on A_n/L_n are free, so distinct elements have Hamming distance one. The resulting metric ultraproduct is therefore the algebraic ultraproduct, and every finite multiplication table in its finitely generated subgroup is realized in one finite factor.

Theorem 2.5 and Proposition 3.3: Expander decomposition and repair of partial intertwiners

Pages 4 and 7–9 · Theorem 2.5 and Proposition 3.3 · arXiv:2608.05362v1

Correct and complete

Kun's theorem supplies an essentially equivalent union of components with one uniform Cheeger constant. For a partial intertwiner $b : Y \dashrightarrow Z$, its graph in the diagonal S-labelled product has boundary bounded by $\Delta_S(b)|Y|$. Removing vertices with incorrect local models costs only the chosen δ-fraction. The cited Kazhdan repair proposition produces a nearby set with small boundary; expansion in the two coordinates then controls empty and multiple rows and columns, allowing it to be corrected to a partial bijection with arbitrarily small source, range, and equivariance defects. The distance estimate retains a constant multiple of the original defect plus the prescribed tolerance.

Verifiable source: Kun–Thom, Proposition 3.3

Lemma 4.2 and Proposition 4.5: The cluster groupoid is well defined and exhausts the centralizer

Pages 11–15 · Lemmas 4.2, 4.4 and Proposition 4.5 · arXiv:2608.05362v1

Correct and complete

Expansion gives the required distance gap: two allowed partial maps are either q_n-close or at least $1 - q_n$ apart. This gap makes improvement-based composition independent of representatives and proves the inverse and associativity axioms. For a centralizing sequence σ_n, the two-sided majority decomposition assigns almost every source component an injective majority target. Markov bounds discard only $o(|X_n|)$ component weight; the repair theorem supplies an allowed arrow on every remaining component, and injectivity lets these arrows be completed within each groupoid component to a total bisection. The resulting permutation is $o(1)$-close to σ_n, proving the nontrivial reverse inclusion.

Proposition 5.1: Uniform flexible stability produces genuine finite actions

Pages 16–17 · Proposition 5.1 · arXiv:2608.05362v1

Correct and complete

Proposition 4.5 supplies maps from the finite groups $F_n = [[C_n]]$ with uniform multiplicative defect tending to zero. Becker–Chapman's theorem applies because every finite F_n is amenable and gives a homomorphism on a set enlarged by only an asymptotically negligible fraction, uniformly close on all of F_n. Uniform asymptotic commutation proves one inclusion in the new centralizer equality. Conversely, a centralizer representative can be altered on the negligible added set to preserve X_n, then Proposition 4.5(c) approximates its restriction by a full-group element; uniform closeness of the corrected action gives the other inclusion.

Verifiable source: Becker–Chapman, Theorem 1.2

Definition 2.1: The multiplicativity variables are omitted after "for all"

Page 3 · Definition 2.1 · arXiv:2608.05362v1

Typo

The printed definition says "for all" and then displays $d_H(\theta_n(\lambda\mu), \theta_n(\lambda)\theta_n(\mu)) \to 0$ without supplying the quantifier. It must read "for all $\lambda, \mu \in \Lambda$." The variables, domain, and intended quantification are uniquely fixed by the displayed formula and by the equivalent free-group formulation immediately below, so this omission does not affect any later argument.

Proof of Theorem A: The essential-equivalence citation omits one item number

Page 17 · final proof, second paragraph · arXiv:2608.05362v1

Typo

The proof cites "Lemma 2.4(ii)" for conjugacy of the embeddings, identification of centralizers, and preservation of ergodicity. The last two conclusions are Lemma 2.4(iii), so the citation should be "Lemma 2.4(ii)–(iii)." Both items were stated and proved available earlier, and the argument uses them correctly.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1
2. Kun–Thom, Proposition 3.3
3. Becker–Chapman, Theorem 1.2

arXiv acknowledgement

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ *is proved for every sufficiently small delta* > 0 *and* P *is immediately monotone in the required direction, then the claim that* $P(\text{delta})$ *holds for every delta* > 0 *requires no correction; the omitted monotonicity step is exposition rather than an audit finding.*

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ *and subsequently applied at* $t = 0$ *remains a formal scope mismatch unless the endpoint case was separately established before the application.*

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.