MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Quantitative Khintchine on the parabola with non-monotonic approximation functions

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Paper	arXiv:2608.04878v1
Source version	Submitted 5 August, 2026; announced August 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	August 5, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	**Statements**	Contains unsupported statements
02	**Proofs**	Contains incorrect or incomplete proofs
03	**Novelty**	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

No counterexample was found to the four quantitative Khintchine statements. Their stated constants are nevertheless not verified: the common proof uses character-sum estimates outside the ranges actually established, and the quantitative lower-envelope reduction does not preserve the displayed dependence on the original approximation function. The restricted-denominator theorem has an additional scope ambiguity.

Theorems 4–5: The all-denominator and large-denominator constants are not established

Pages 2–3 and 7–15 · Theorems 4–5 and Sections 3.1–3.2 · arXiv:2608.04878v1

Not able to verify

Both conclusions depend on Lemma 17 for every partial-sum length $N = J/d$. In its small-modulus branch, however, the proof invokes Lemma 9, which is established only when $q_1^{3/8} \le N \le q_1^{5/8}$; neither the definition $J = \lfloor (6\kappa\psi(q))^{-1} \rfloor$ nor the theorem hypotheses impose this range. The proof also replaces ψ by $\widetilde{\psi}(q) = \max\{\psi(q), q^{-5/8+\eta}\}$, thereby increasing S_ψ and possibly $\max \psi$, but it continues to use the admissible bound for κ computed from the original quantities. These are genuine quantitative obligations, so the printed argument does not prove the displayed constants. They do not furnish a counterexample to either theorem.

Verifiable source: Paper, version 1

Theorem 6: The restricted-prime-factor theorem is not verified in its printed scope

Pages 3 and 15–16 · Theorem 6 and Section 3.3 · arXiv:2608.04878v1

Not able to verify

The proof begins with $d(q) \le 2^n$ because q has at most n prime divisors. Under the standard reading that "prime divisors" means distinct prime divisors, this is false: $q = 2^m$ has one prime divisor but $d(q) = m + 1$. The bound is valid if the hypothesis is instead $\Omega(q) \le k$ (prime factors counted with multiplicity), or if q is squarefree with at most k prime factors. The theorem must state which restriction is intended and propagate it consistently. Independently, it inherits the Lemma 17 range gap and the quantitative lower-envelope gap from Theorem 4. No counterexample to the measure conclusion was found.

Verifiable source: Paper, version 1

Theorem 7: The prime-denominator constant is not established for all admissible κ

Pages 3 and 16–17 · Theorem 7 and Section 3.4 · arXiv:2608.04878v1

Not able to verify

The proof applies Lemma 12 to every partial sum $\sum_{j \le u} (j/p)$ for $1 \le u \le J$. The cited bound is available only for interval length $u \le p^{5/8}$, while the theorem permits arbitrarily small positive κ and hence arbitrarily large $J = \lfloor (6\kappa\psi(p))^{-1} \rfloor$. Splitting off the multiples of p does not put the remaining partial sums into the cited range; no estimate for the uncovered lengths is supplied. The theorem also inherits the unpreserved lower-envelope reduction. This prevents verification of the stated constant but does not disprove the conclusion.

Verifiable source: McGown, Theorem 7.1 source

Theorems 4–7 · zero function: The identically zero approximation function needs a separate clause

Pages 2–3 · hypotheses and displayed bounds for $\kappa(\delta)$ · arXiv:2608.04878v1

Minor formal correction

Each theorem allows $\psi \equiv 0$, but its displayed bound then contains division by $S_\psi = 0$ and by $\max \psi = 0$. Add the assumption $S_\psi > 0$, together with a separate sentence that $\psi \equiv 0$ gives $\mathcal{B}'(\psi, \kappa) = [0, 1]$ for every positive κ. This boundary repair is immediate and does not affect any nonzero approximation function.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proof does not justify the quantitative reduction to a power-law lower envelope, the small-modulus character estimate is used beyond its proved range and for a second character not checked by the supplied computation, and the prime proof likewise exceeds the cited Burgess range. The imprimitive-character passage is also incorrect as written. Several local formula and endpoint defects have verified repairs and are reported separately in yellow.

Section 3.1 · lower-envelope reduction: Replacing ψ changes the quantities that determine the advertised constant

Page 7 · first paragraph of Section 3.1; used throughout Sections 3.1–3.4 · arXiv:2608.04878v1

Incomplete as written

The inclusion $\mathcal{B}'(\widetilde{\psi}, \kappa) \subseteq \mathcal{B}'(\psi, \kappa)$ is correct, but it is insufficient for the quantitative theorem. The replacement $\widetilde{\psi}(q) = \max\{\psi(q), q^{-5/8+\eta}\}$ increases S_ψ and can increase $\max \psi$. The proof's final admissible bounds for κ are decreasing functions of those quantities, so a κ allowed by the theorem for the original ψ need not satisfy the bound derived for $\widetilde{\psi}$. The coefficient-one lower bound is then used essentially to dominate three weighted series by S_ψ. Repair classification: No repair supplied; a quantitative approximation or a direct split of the small values of ψ must preserve all constants. For Theorems 5–7 the replacement must also preserve their support restrictions.

Lemmas 9 and 17: The small-modulus bound is applied for unproved lengths and an unchecked character

Pages 5, 9–12, and 17–19 · Lemma 9, Lemma 17, Equation (7), and Appendix A · arXiv:2608.04878v1

Incomplete as written

Lemma 9 is stated only for $q^{3/8} \le N \le q^{5/8}$, but Lemma 17 asserts its resulting $\sqrt{N} q_1^{11/16}$ estimate for every N, and Equation (7) uses it at $N = u$ throughout $1 \le u \le J/d$. No argument treats $u > q_1^{5/8}$, which occurs for sufficiently small κ. Moreover, Appendix A computes only 'kronecker_symbol(N,q)', corresponding to (N/q); it does not verify the separately stated character (q/N) needed when $q_1 \equiv 0 \pmod 4$. Repair classification: No repair supplied; the missing length ranges and the second character must be proved or exhaustively certified before Lemma 17 can support the main estimates.

Remark 11: The imprimitive-character reduction is not a valid derivation as printed

Page 6 · Remark 11 and Equation (4) · arXiv:2608.04878v1

Incorrect as written

An induced character is printed as $\chi = \chi_1\chi_2$ with primitive and principal moduli satisfying $k_1 + k_2 = q$; no such additive modulus relation gives the displayed character identity. After writing $n = ut$, the formula also silently replaces $\chi_1(ut)$ by $\chi(u)$ and omits the factor $\chi_1(t)$. Absolute values can remove a nonzero factor $\chi_1(t)$, but they do not justify replacing the primitive character by the original imprimitive one. Finally, the applicable Burgess estimate is governed by the conductor k_1, not merely by which numerical range contains q. Repair classification: Plausible repair only. The standard conductor/radical decomposition can likely yield a divisor-factor loss, but its cases and constants must be written and propagated to Lemma 17.

Verifiable source: Standard induced-character decomposition

Section 3.4 · prime character sums: McGown's interval-length hypothesis is not checked

Page 17 · application of Lemma 12 before the final measure estimate · arXiv:2608.04878v1

Incomplete as written

Lemma 12 provides the constant 10.0366 only for lengths $N \le p^{5/8}$. Abel summation requires the bound for every $u \le J$, but no inequality $J \le p^{5/8}$ follows from the hypotheses; indeed J grows without bound as κ decreases. Periodicity cancels complete periods, yet a remaining interval can still have length between $p^{5/8}$ and $p - p^{5/8}$, and the paper supplies no estimate there with the displayed constant. Repair classification: No repair supplied; an all-length estimate or a separate treatment of the remaining interval lengths is required.

Verifiable source: McGown, Norm-Euclidean cyclic fields of prime degree

Theorem 6 · divisor count: The proof needs prime factors counted with multiplicity

Page 15 · first sentence of Section 3.3 · arXiv:2608.04878v1

Incomplete as written

The estimate $d(q) \le 2^n$ follows when q is a product of at most n primes counted with multiplicity: if $q = \prod p_i^{a_i}$ and $\sum a_i \le n$, then $d(q) = \prod(a_i + 1) \le 2^{\sum a_i}$. It does not follow from having at most n distinct prime divisors. The proof therefore verifies only the narrower interpretation and must state it explicitly. Repair classification: Verified conditional repair; replace the support hypothesis by $\Omega(q) \le k$ (or impose squarefreeness with $\omega(q) \le k$) and replace the stray n by k throughout.

Fejér kernel: The closed form has an extra factor of two

Page 7 · display defining $\mathcal{F}_J$ and the following local estimate · arXiv:2608.04878v1

Typo

For the printed Fourier series, the correct identity is

$$\mathcal{F}_J(x) = \frac{1}{J^2}\left(\frac{\sin(\pi J x)}{\sin(\pi x)}\right)^2,$$

not the displayed formula with $2\pi Jx$ and $2\pi x$. For example, at $J = 2, x = 1/4$ the Fourier series equals $1/2$ while the printed quotient equals 0. Replacing both occurrences of 2π by π also makes the next local-sine argument valid and preserves the claimed lower bound $4/\pi^2$. The Fourier expansion determines this correction uniquely, so no subsequent constant changes.

Verifiable source: Paper, version 1

Initial covering step: One scale factor and the endpoint representatives need correction

Page 7 · displays immediately preceding Equation (5) · arXiv:2608.04878v1

Minor formal correction

From $|qx - a|, |qx^2 - b| \le \kappa\psi(q)$ one obtains

$$\left|a^2/q^2 - b/q\right| \le 3\kappa\psi(q)/q,$$

so the displayed right side is missing '/q'; multiplying by q then gives the correctly used condition $\|a^2/q\| \le 3\kappa\psi(q)$. Also, near $x = 0$ the nearest numerator can be $a = 0$, contrary to the printed $1 \le a \le q$. Include $a = 0$ and pair the two endpoint half-intervals, or state their measure separately; because $a = 0$ and $a = q$ represent the same residue, their two half-lengths equal the one full interval already charged in Equation (5). These repairs are local and leave that measure bound unchanged.

Theorem 5 and Theorem 6 notation: Several symbols have unique mechanical corrections

Pages 3, 14–15 · Theorem 6 and the Cauchy–Schwarz display in Section 3.2 · arXiv:2608.04878v1

Typo

In Theorem 6 and its proof, the undefined n in 2^{-n} and $2^{7n/2}$ should be the theorem's parameter k. In the first Cauchy–Schwarz line on page 15, replace $\sum \psi(q)$ by $\sum \psi(q)^2$ and $(r^2 t)^2$ by $(r^2 t)^{-2}$; the immediately following line already uses exactly these corrected expressions. Finally, in the preceding Theorem 5 divisor split, the last nonsquare condition should cover all $q_1 > 10^5$; the constant 27 dominates the large-range constant 5, and the subsequent displayed count is already over $q_1 > 10^5$. Each correction is forced by the adjacent formulas and changes no downstream estimate.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2608.04878v1
2. Paper, version 1
3. McGown, Norm-Euclidean cyclic fields of prime degree
4. Standard induced-character decomposition

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ *is proved for every sufficiently small delta* > 0 *and* P *is immediately monotone in the required direction, then the claim that* $P(\text{delta})$ *holds for every delta* > 0 *requires no correction; the omitted monotonicity step is exposition rather than an audit finding.*

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ *and subsequently applied at* $t = 0$ *remains a formal scope mismatch unless the endpoint case was separately established before the application.*

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.