MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Strichartz estimates for quasi-periodic functions on long time intervals: An arithmetic approach

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Prompt	Standardized prompt
Novelty cutoff	August 4, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The long-time Strichartz estimates for quadratic and higher-degree algebraic frequency ratios, their torus reformulation, and the endpoint fractional estimate are correct. The degree-dependent time scales follow from the verified lattice-energy counts, and the endpoint loss follows from the cited reverse square-function theorem together with the height and frequency decompositions.

Theorem 2: Quadratic-frequency long-time estimate

Page 4 · Theorem 2, Equation (5) · arXiv:2608.03194v1

Correct

For an algebraic irrational ω of degree 2, the sixth-moment expansion reduces to counting triples in $\mathbb{Z} + \omega\mathbb{Z}$ with fixed sum and nearly fixed sum of squares. Lemma 6 gives C^{ε} choices at each exact energy, while Roth's theorem gives C^{ε} admissible energies in a window of width C^{-2}. The remaining dyadic energies contribute the same $C^{2+\varepsilon}$ sixth-power bound after time integration. Thus the estimate on an interval of length C^2 has factor $C^{1/3+\varepsilon}$; interpolation with the height-Bernstein bound and subdivision of $[0, T]$ give exactly

$$\left(1 + \left(\frac{T}{C^2}\right)^{1/p}\right) C^{1-4/p+\varepsilon}.$$

Verifiable source: Full paper, version 1

Theorem 3: Higher-degree long-time estimates

Page 5 · Theorem 3, Equations (6)–(7) · arXiv:2608.03194v1

Correct

When $\deg \omega > 2$, the numbers $1, \omega, \omega^2$ are linearly independent over $\mathbb{Q}$. The multivariate Roth bound therefore gives $C^{2+\varepsilon}$ possible near-resonant energies at scale C^{-2} and only C^{ε} at scale C^{-4}. These are precisely the losses required for the two regimes in Equation (6). At the fourteenth moment, the Bourgain–Demeter quadratic Parsell–Vinogradov estimate and the same Diophantine count yield

$$C^{\varepsilon}\left(C^{5/7} + T^{1/14} C^{3/7}\right),$$

and interpolation with the supremum norm gives Equation (7) for every $p \geq 14$.

Verifiable source: Bourgain–Demeter, quadratic Parsell–Vinogradov estimate

Theorem 6: Endpoint fractional Strichartz estimate

Page 6 · Theorem 6, Equation (8) · arXiv:2608.03194v1

Correct

At frequency scale N, the cited reverse square-function theorem partitions the curve at horizontal scale N^{-1} when $a \geq 2$ and $N^{-a/2}$ when $0 < a < 2$, $a \neq 1$. A corresponding frequency slab contains at most $O_{\vec{\omega}}(C^{\nu-1})$ lattice points in the first case and $O_{\vec{\omega}}(N^{1-a/2}C^{\nu-1} + 1)$ in the second. The L^2-to-L^4 Bernstein estimate therefore gives the factor

$$C^{(\nu-1)/4}\left(1 + N^{\sigma_a}\right), \qquad \sigma_a = \begin{cases} 0, & a \geq 2, \\ (2-a)/8, & 0 < a < 2,\ a \neq 1. \end{cases}$$

Frequency and height square-function summation then produces the stated Sobolev exponent $(\nu - 1)/4 + \sigma_a$, including the endpoint.

Verifiable source: Bulj–Inami–Shiraki, Theorem 1.2

Corollary 1: Degenerate torus reformulation

Pages 5–6 · Corollary 1 · arXiv:2608.03194v1

Correct

Lemma 1 identifies the spatial mean norm of a two-frequency quasi-periodic polynomial with the corresponding norm on $\mathbb{T}^2$. Under this identification, the phase $(k_1 + \omega k_2)^2$ is exactly the symbol of the displayed rank-one operator $\widetilde{\Delta}$. Substitution of Theorems 2 and 3 gives all three clauses with the printed powers of T and C.

02 / PROOFS

Proofs

Correct

The central proof chains are correct and complete after the uniquely determined typographical repairs listed below. The exact-energy lemma reduces to divisor and oval lattice-point counts; the higher-degree argument uses the stated Schmidt-subspace estimate with the correct separation exponents; the fourteenth-moment input follows from Bourgain–Demeter decoupling; and the endpoint argument applies the reverse square-function theorem at its supported scales.

Lemma 6 and proof of Theorem 2: Exact and near-energy counting

Pages 12–16 · Lemma 6 and proof of Theorem 2 · arXiv:2608.03194v1

Correct and complete

For $\deg \omega > 2$, comparison of the coefficients of $1, \omega, \omega^2$ reduces the exact fiber to two positive-definite Eisenstein norm equations, each having C^ε representations. In the quadratic case, the corrected identities recorded in the typo finding below put $(Q(N), \Delta(K, N))$ on a fixed dilate of an ellipse; the cited oval estimate gives C^ε possibilities, the norm equation gives C^ε choices of N, and the bilinear equation together with Δ determines K. Roth separation then bounds the number of nearby energy values. The same separation applied to differences of two admissible coefficient pairs supplies the elementary packing step compressed on pages 14 and 16.

Verifiable source: Bombieri–Pila, integral points on analytic ovals

Proof of Theorem 3 for $6 \le p < 14$: Two Diophantine time scales

Pages 17–19 · Section 5.1 · arXiv:2608.03194v1

Correct and complete

Writing an energy as $t_1 + \omega t_2 + \omega^2 t_3$, with $|t_i| \lesssim C^2$, the multivariate Roth inequality makes distinct coefficient triples in a window of width C^{-2} separated by $C^{2/(2+\varepsilon)}$ and those in a window of width C^{-4} separated by $C^{4/(2+\varepsilon)}$. Packing in the two unconstrained coefficient directions gives respectively $C^{2+\varepsilon}$ and C^ε values. Lemma 6 controls every exact fiber, and the dyadic nonresonant sums have the required $AC^{4+\varepsilon}$ bound. Taking sixth roots, interpolating, and subdividing time yields both regimes of Equation (6).

Proof of Theorem 3 for $p \ge 14$: Parsell–Vinogradov and restricted-input reduction

Pages 19–21 · Section 5.2 · arXiv:2608.03194v1

Correct and complete

The paper's inhomogeneous subset estimate

$$J_s(N, S) \lesssim_{\varepsilon,s} N^\varepsilon \left(N^{2s-2} + N^{4s-10}\right) |S|$$

follows from the cited Bourgain–Demeter discrete restriction estimate: for $s \le 4$ its moment bound is $N^\varepsilon |S|^s \le N^{2s-2+\varepsilon}|S|$, while for $s \ge 4$ it is $N^{2s-8+\varepsilon}|S|^s \le N^{4s-10+\varepsilon}|S|$. A shifted right-hand side is a Fourier coefficient of the same nonnegative moment and is bounded by the zero coefficient. After six of the fourteen variables are fixed, the case $s = 4$ gives $C^6|S|^7$. Counting the remaining energy coefficients at each dyadic scale produces $C^\varepsilon(C^{10} + TC^6)|S|^7$, whose fourteenth root is the claimed estimate. Lemma 7 then transfers the restricted estimate to arbitrary coefficients.

Verifiable source: Bourgain–Demeter, Theorems 1.1–2.2 and Corollary 2.3

Proposition 2 and proof of Theorem 6: Reverse square-function application and dyadic summation

Pages 21–24 · Proposition 2 and proof of Theorem 6 · arXiv:2608.03194v1

Correct and complete

After the rescaling $(x,t) \mapsto (N^{-1}x, N^{-a}t)$, the thickening of the curve has width comparable to N^{-a}. The cited theorem applies with partition exponent $b = a$ for $a \geq 2$ and $b = 2$ for $0 < a < 2$, so its loss exponent is zero in both cases. Finite-overlap enlargement of the projected slabs, the vector-valued Hilbert-transform bound, and the weighted spatial-mean identity reduce the estimate to lattice points in a single slab. The elementary choice of one coordinate with nonzero ω_j verifies the stated slab counts. Plancherel and the two Littlewood–Paley summations then give exactly Equation (8). Frequencies $N < 1$, for which the displayed rescaling is unnecessary, follow directly from the same L^2-to-L^4 slab bound and introduce no additional loss.

Verifiable source: Bulj–Inami–Shiraki, reverse square-function estimate

Lemma 6, quadratic cases: Two scale factors in the conic identities are extraneous

Pages 12–13 · proof of Lemma 6, Cases 2-1 and 2-2 · arXiv:2608.03194v1

Typo

In Case 2-1 the displayed identity has an extra factor a before the first square. From the two preceding equations and $4Q(K)Q(N) - L_2^2/4 = 3\Delta(K,N)^2$, the correct identity is

$$(4cQ(N) + L_1)^2 - 12ac\Delta(K,N)^2 = L_1^2 + acL_2^2.$$

In Case 2-2, with $D = b^2 - 4ac$ and $A = 2aL_1 - bL_2$, the right-hand side must be $A^2 - DL_2^2$, not $DA^2 - DL_2^2$. Direct expansion uniquely determines both corrections. Each corrected equation is still an ellipse equation, so the lattice-point argument is unchanged.

Proof of Theorem 2: The coefficient norm is omitted from two intermediate target displays

Pages 14 and 16 · Step 1 after the near-resonant integral and Step 2 before Equation (21) · arXiv:2608.03194v1

Typo

The first “it is enough to show” display ends with C^ε, and the later display ends with $AC^{2+\varepsilon}$, without the homogeneous factor. In both places the right-hand side must also contain

$$\left(\sum_\lambda |a_\lambda|^2\right)^3.$$

The immediately following Cauchy–Schwarz displays contain exactly this factor and prove the corrected inequalities, so the omission is local and harmless.

Proof of the fourteenth-moment estimate: One occurrence of the quadratic energy drops the exponent

Page 21 · display immediately after the dyadic energy decomposition · arXiv:2608.03194v1

Typo

The counting display prints $|\alpha + 2\beta\omega + \gamma\omega| \sim 2^j$. It must read $|\alpha + 2\beta\omega + \gamma\omega^2| \sim 2^j$, as in the preceding and following displays and as dictated by $(k_1 + \omega k_2)^2$. This unique correction restores the linear form to which the multivariate Roth estimate is applied.

Proposition 2, Equation (28): The time cutoff and time domain are mismatched in one square-function display

Page 23 · Minkowski display and Equation (28); repeated in the second case on page 24 · arXiv:2608.03194v1

Typo

The line obtained from Minkowski drops the factor $\eta(t)$ even though it occurs immediately before and after that line; it must be retained. Correspondingly, the right-hand norm in Equation (28) is printed over $[0,1] \times \mathbb{R}$ although it contains $\eta(t)$ and the adjacent formulas use $\mathbb{R} \times \mathbb{R}$. Its time domain must be $\mathbb{R}$. With these mechanically determined corrections, the subsequent Bernstein estimate is on the same domain and the original $[0,1]$ estimate follows because $|\eta|$ is bounded below there.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1
2. Bourgain–Demeter, Theorems 1.1–2.2 and Corollary 2.3
3. Bulj–Inami–Shiraki, reverse square-function estimate
4. Bombieri–Pila, integral points on analytic ovals

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.