MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Freidlin-Wentzell collision-laws between self-stabilizing diffusions

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Paper	arXiv:2607.27932v2
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Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	August 1, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

No counterexample to either principal collision law was established. Both remain unverified under the printed assumptions, however. The proof applies the paper's bounded-domain, globally-Lipschitz Freidlin–Wentzell theorem to unbounded domains and polynomial drifts, and its exponentially long coupling uses a local quadratic stability property that does not follow from asymptotic stability. The particle theorem also requires a population threshold depending on the collision radius, while its statement quantifies over one fixed sufficiently large population before taking the radius to zero.

Theorem 1.5: Collision law for the two self-stabilizing diffusions

Pages 9 and 13–28 · Theorem 1.5 and Sections 2–3 · arXiv:2607.27932v2

Not able to verify

The action H_0 is the natural candidate for the iterated small-radius and small-noise collision cost, and the finite-cover argument would transfer fixed-center hitting laws to the first collision. Two indispensable inputs are not established under Assumptions (A), though. First, Lemma 2.2 invokes Theorem 1.4 outside the theorem's printed bounded-domain and globally-Lipschitz hypotheses. Second, Proposition 3.2 assumes local quadratic attraction of $-U$ at each well, whereas (A)-(ii) gives only Lyapunov asymptotic stability. Since these inputs control the process for times of order $\exp(2H/\sigma^2)$, the displayed double limit and persistence conclusion are not verified by the present proof. This is not a claim that the theorem is false.

Theorem 1.6: Collision law for a fixed sufficiently large particle population

Pages 10 and 29–35 · Theorem 1.6 and Section 4 · arXiv:2607.27932v2

Not able to verify

This theorem inherits the unresolved Freidlin–Wentzell and local-stability inputs from Theorem 1.5. It also states the iterated limit for every fixed N above one unspecified threshold. Proposition 4.2 instead produces $N_{\varepsilon,\kappa}$ depending on the collision radius and coupling tolerance; the transfer to a radius-ε hitting event requires the tolerance to be smaller than ε. No uniform bound on $N_{\varepsilon,\kappa}$ as $\varepsilon \downarrow 0$ is proved. The argument therefore verifies, at most, a formulation in which N is chosen after fixing ε, not the printed quantifier order.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The finite-cover reductions and many coupling estimates are internally consistent after local notation corrections, but three central obligations remain: extending the exit theorem to the unbounded polynomial-drift setting actually used, justifying exponentially long localization from the stated stability assumption, and making the particle-rank choice uniform in the outer radius limit. The printed definition of the positive small-radius threshold also uses an infimum where its own explanation requires a supremal interval.

Lemma 2.2 and Lemma 2.4: The recalled exit theorem is applied outside two explicit hypotheses

Pages 8 and 13–17 · Theorem 1.4 and Lemmas 2.2–2.4 · arXiv:2607.27932v2

Incomplete as written

Theorem 1.4 in the paper assumes an open bounded exit domain and globally Lipschitz drift and diffusion. Lemma 2.2 applies it to $G = (D^1_{\lambda,\varepsilon} \times D^2_{\lambda,\varepsilon})^c$ and its modified versions. These domains are generally unbounded; for the one-dimensional contractive drift $-\Psi_i(x) = -(x - \lambda_i)$, each complement $(D^i_{\lambda,\varepsilon})^c$ is already an unbounded half-line when the target ball misses λ_i. Moreover, the main drifts $W_{\lambda_i} = U + F(\mathring{u} - \lambda_i)$ have the polynomial growth allowed by (A)-(i) and (A)-(iii), not global Lipschitz continuity. No extension theorem or truncation-and-outer-boundary argument is supplied. Downstream dependency: Lemma 2.4, Propositions 2.5, 3.3–3.4, and both main theorems. A repair must prove an unbounded-domain exit law under the paper's coercive polynomial hypotheses or truncate the domains and show uniformly that every artificial outer boundary has strictly larger action.

Proposition 3.2, Step 4: Asymptotic stability is used as if it implied quadratic local coercivity

Pages 26–27 · Step 4 of Proposition 3.2 · arXiv:2607.27932v2

Incomplete as written

The proof asserts that the Jacobian of $-U$ at λ_1 is negative definite and hence that, for some $\rho, r > 0$, $B(\lambda_1; r)$ lies in

$$S_\rho = \{x : (x - \lambda_1) \cdot U(x) \geq \rho \|x - \lambda_1\|^2\}.$$

Assumption (A)-(ii) states only Lyapunov asymptotic stability. That does not imply hyperbolicity or this quadratic bound: the scalar flow $\dot{x} = -x^3$ is asymptotically stable at zero but satisfies $xU(x) = x^4$, which cannot dominate ρx^2 near zero. The other assumptions do not add the asserted property for U itself. This bound is then used to keep $\mathbb{E}\|X_t - \lambda_1\|^2$ small up to exponentially large times. A verified sufficient repair is to assume explicitly that $(x - \lambda_i) \cdot U(x) \geq \rho_i \|x - \lambda_i\|^2$ in a neighborhood of each well, or replace Step 4 by a localization argument valid for nonhyperbolic stable equilibria.

Theorem 1.6 and Proposition 4.2: The population threshold is not uniform in the radius limit

Pages 10 and 29–35 · Theorem 1.6, Proposition 4.2, and Section 4.2 · arXiv:2607.27932v2

Incomplete as written

Theorem 1.6 first fixes a sufficiently large N and then takes $\varepsilon \downarrow 0$. Proposition 4.2 chooses $N_{\varepsilon,\kappa}$ after ε and κ, and the bracketing of radius-ε hitting times requires $\kappa < \varepsilon$. The proof gives no bound on $\sup_{0<\varepsilon<\varepsilon_*} N_{\varepsilon,\kappa(\varepsilon)}$. Repair classification: unresolved under the printed theorem. Either establish such a uniform bound, or change the conclusion to state the fixed-ε law with $N \geq N(\varepsilon)$ and specify a joint order of limits.

Equations defining ε_c: An infimum cannot define the advertised largest small-radius interval

Pages 17 and 27 · definitions preceding Propositions 2.5 and 3.4 · arXiv:2607.27932v2

Typo

Both definitions print ε_c as the infimum of radii at which the desired strict comparison holds, then call it the largest radius and use the comparison for every $0 < \varepsilon < \varepsilon_c$. If the comparison holds for all sufficiently small radii, the printed infimum is zero. The definition dictated by the following prose is

$$\varepsilon_c = \sup\{r > 0 : \text{the comparison holds for every } \varepsilon \in (0, r)\}.$$

This symbol-level correction does not change the intended results, but the proof must still verify that this set contains a positive interval.

Assumptions and coupling notation: Four local symbols conflict with their immediately stated meaning

Pages 5, 22, and 24 · (A)-(i), (A)-(ii), and Proposition 3.2 · arXiv:2607.27932v2

Typo

The strong-convexity inequality in (A)-(i) is printed with $\leq$ although the subsequent deductions require $\xi \cdot \nabla U(x)\xi \geq C\|\xi\|^2$; (A)-(ii) writes $B(\lambda_1;\varepsilon)$ for both wells instead of $B(\lambda_i;\varepsilon)$. In Proposition 3.2, $S_\kappa^{(1)}$ and the force estimate use $\mathbb{E}\|X_{T_\kappa} - \lambda_1\|^2$ where their dependence on the running time and every later line require $\mathbb{E}\|X_t - \lambda_1\|^2$. These replacements are uniquely determined by the surrounding logic and are treated as typos, not adverse statement findings.

Proposition 4.2, Step 5.2.3: A product probability is used despite interacting particles

Page 33 · Step 5.2.3 of Proposition 4.2 · arXiv:2607.27932v2

Incorrect as written

The proof bounds the probability that at least cN tagged couplings fail by a binomial sum involving $\nu(\sigma)^\ell$. The failure events are not independent because the $X^{i,N}$ interact through their empirical measure, so the product power is unjustified. Repair classification: verified repair. Exchangeability and Markov's inequality give

$$\mathbb{P}\left(\frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{A_i} \geq c\right) \leq \frac{\mathbb{E}[N^{-1}\sum_i \mathbf{1}_{A_i}]}{c} = \frac{\mathbb{P}(A_1)}{c} \longrightarrow 0,$$

which supplies exactly the high-probability conclusion needed in the following step.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2607.27932v2

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.