MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Perfect 2-codes over arbitrary alphabets

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Paper	arXiv:2607.27555v1
Source version	Submitted 30 July, 2026; announced July 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	July 30, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains unverified proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The qualitative bounded-prime-factor finiteness results are correct, and Theorem 5 independently verifies the qualitative finiteness portion of Theorem 2. No counterexample was found to the explicit exclusions in Theorems 2 and 4. Those assertions are nevertheless not able to be verified because they depend on several large exhaustive computations for which the paper supplies neither code nor independently checkable certificates.

Theorem 2, qualitative clause: Finiteness for alphabets of size $2^\alpha p^\beta$

Page 2 · First sentence of Theorem 2; independently implied by Theorem 5 on pages 2 and 16–17 · arXiv:2607.27555v1

Correct

Theorem 5 applies with $P_0 = 2$ and $q_0 = 2^\alpha$, and therefore gives only finitely many perfect 2-codes with alphabet size $q = 2^\alpha p^\beta$. Its proof bounds the exponent of the exceptional prime through Matveev's theorem and then uses Ridout's theorem for the finitely many remaining algebraic targets. After this leaves only finitely many exceptional primes, Theorem 3 bounds the remaining alphabet sizes. This verifies the qualitative finiteness clause independently of the computations used earlier in Section 3.

Verifiable source: Full paper, version 1

Theorem 2, explicit necessary conditions: The exclusions $\alpha \leq 20$ and $p \leq 10^{10}$ are not independently verified

Page 2 · Second sentence of Theorem 2; pages 10 and 13–15 · Proposition 3.4 and Section 3.5 · arXiv:2607.27555v1

Not able to verify

The exact claim is that any such code must have $\alpha > 20$, $\beta \leq 2519$, $p > 10^{10}$, and $p \equiv 3 \pmod 8$. The bounds on β and the congruence condition follow from the displayed analytic and elementary arguments. The lower bound on p rests on a scan of 455,052,509 primes, and the exclusion of every $\alpha \leq 20$ rests on Hensel-lifting scans over more than sixty million parameter triples. The paper gives only the asserted outputs, without code, complete output, hashes, or a certificate from which every case can be checked. Those two indispensable computational conclusions could not be independently reproduced in this audit. No counterexample to the theorem was found.

Verifiable source: Full paper, version 1

Theorem 3: Effective finiteness for bounded greatest prime factor

Pages 2 and 5 · Theorem 3 and its proof · arXiv:2607.27555v1

Correct

Proposition 2.1 makes the two Lloyd roots unusually close, while both are composed only of primes dividing $2q$. Tijdeman's effective gap theorem for integers with bounded prime factors supplies the opposing lower bound $r_2 - r_1 > r_1/(\log r_1)^\kappa$, with κ depending only on $P(q)$. Combining the bounds gives $r_1 q < 6(\log r_1)^{2\kappa}$, which bounds r_1, then q and r_2, in terms of the fixed prime bound. Reuvers's fixed-q finiteness theorem then gives the stated conclusion.

Verifiable source: Tijdeman, On integers with many small prime factors

Theorem 4: The complete elimination for $P(q) \leq 13$ is not independently verified

Pages 2 and 5–6 · Theorem 4 and its proof · arXiv:2607.27555v1

Not able to verify

The cited de Weger theorem reduces the argument to 605 coprime pairs of 13-smooth integers. The manuscript then asserts that inequalities (11)–(12) eliminate all but eight listed pairs and that direct substitution eliminates those remaining cases. The reductions and the final substitutions are clear, but the full list of 605 pairs and the intermediate elimination data are not supplied, so the decisive exhaustive check cannot be reconstructed from the paper. No surviving parameter tuple or counterexample to Theorem 4 was found.

Verifiable source: Full paper, version 1

Theorem 5: Finiteness with one unrestricted prime factor

Pages 2 and 16–17 · Theorem 5 and Section 4 · arXiv:2607.27555v1

Correct

After removing the common smooth factor of the two Lloyd roots, the largest remaining prime power is at least the power of the unrestricted prime. Proposition 2.1 and Matveev's lower bound therefore bound its exponent uniformly in terms of P_0. For each of the finitely many exponents and residue classes of the smooth exponents, inequality (47) is a Ridout approximation to one fixed algebraic number. The lower bound for q_2 verifies the required numerator restriction for $\mu = 1/3$, $2/3$, or 1 according as $\gamma = 1$, 2, or at least 3, while the denominator is supported on the fixed small-prime set. Ridout leaves finitely many exceptional primes, and Theorem 3 then completes the finiteness argument.

Verifiable source: Ridout, Rational approximations to algebraic numbers

02 / PROOFS

Proofs

Contains unverified proofs

The qualitative arguments for Theorems 3 and 5 are correct and complete. The explicit parts of Theorems 2 and 4 depend on three finite computations whose outputs are asserted but not supplied in a reproducible or certificate-checkable form. Four mechanically determined numerical or notation defects are reported in yellow and do not themselves change the conclusions.

Proposition 3.4: The exhaustive continued-fraction scan is not verifiable from the supplied material

Page 10 · Computation following Equation (26) · arXiv:2607.27555v1

Not able to verify

The proof requires checking the first twenty continued-fraction terms of $\log p/\log 2$ for every one of the 455,052,509 primes $5 \leq p < 10^{10}$ and asserts that only $p = 5$ and $p = 181$ pass Equation (26). The search algorithm is described at a high level, but no implementation, precision policy, interval-arithmetic check, complete candidate output, or independently checkable certificate is included. This leaves a precise nontrivial obligation: verify that no omitted prime has a convergent with $2 \leq q_i \leq 5039$ satisfying Equation (26), and then verify the two residual substitutions. Proposition 3.4, and hence the explicit condition $p > 10^{10}$ in Theorem 2, is not able to be verified from the available evidence.

Verifiable source: Full paper, version 1

Section 3.5: The two Hensel-lifting eliminations have no checkable certificate

Pages 13–15 · Paragraphs following Equations (37) and (43) · arXiv:2607.27555v1

Not able to verify

For $2 \leq \alpha \leq 20$, the proof must show for every admissible (γ, α, β) that a lifted residue r_j exceeds the bound in Equation (35) while $2^j \cdot 100 < 33\gamma$. For $\alpha = 1$, it must do the analogous check for every admissible (γ, β, ω), including the branch where $f'(x) \equiv 2 \pmod 4$. The manuscript reports that all cases are eliminated and gives one example, but supplies neither the code nor per-case residues or a compact certificate. The special even-derivative lifting procedure is also not stated. The necessary all-cases assertions therefore remain unverified, and they are exactly the input used to conclude that no code exists for $\alpha \leq 20$.

Verifiable source: Full paper, version 1

Proof of Theorem 4: The 605-pair elimination is asserted without its finite data

Pages 5–6 · Application of Equations (11)–(12) after Theorem 6 · arXiv:2607.27555v1

Not able to verify

The proof says that the inequalities eliminate 597 of de Weger's 605 pairs, leaving the eight displayed pairs and small ranges for d. The paper does not provide the list or the calculated bounds for the omitted 597 cases. The final substitutions for the stated residual cases are verifiable, but the exhaustive reduction to that residual list is not. Repair classification: No repair supplied; a table, code with exact arithmetic, or a compact certificate covering all 605 inputs would make the step checkable.

Verifiable source: Full paper, version 1

Equation (24): The final exponent of p is twice the intended exponent

Page 10 · Equation (24) · arXiv:2607.27555v1

Typo

The printed last bound is $1/(2\gamma p^{\gamma})$, which does not follow from the middle expression. Replace it by $1/(2\gamma p^{\gamma/2})$. Indeed, $q \geq 94$ makes the remaining constant smaller than $1/2$, and $p^{\gamma/2} \geq \gamma$ then gives the $1/(2\gamma^2)$ bound needed for Legendre's criterion. Equation (26) already uses the corrected half-exponent through $p^{q_i/2}$, so this unique correction is mechanical and does not change the argument.

Verifiable source: Full paper, version 1

Proposition 3.5 induction: The induction hypothesis cites the wrong equation

Page 11 · Proof of Proposition 3.5, sentence before the display defining M · arXiv:2607.27555v1

Typo

The proof says, "Assume that we have (14)" for a fixed k. Replace "(14)" by "(28)": the next display is exactly Equation (28) with an added multiple $Mp^{(k+1)\beta}$, and Equation (14) is then separately substituted on the following line. The intended reference is unique and no calculation changes.

Verifiable source: Full paper, version 1

Reported parameter counts: Two totals include one extraneous parameter slice

Pages 13 and 15 · Counts preceding the Hensel-lifting computations · arXiv:2607.27555v1

Typo

For the ranges printed on page 13, namely odd $5 \leq \gamma \leq 5039$, $2 \leq \alpha \leq 20$, and $1600\beta^2 < 1089\gamma$, exact integer counting gives 1,845,603 triples, not 1,942,740; the printed total is obtained by also allowing the extraneous value $\alpha = 1$. On page 15, the listed $\omega \geq 2$, $\gamma \geq 9$ range contains 61,634,388 triples, not 61,634,419; the difference is exactly the 31 extraneous cases $(\gamma, \beta) = (8, 1)$ with $2 \leq \omega \leq 32$. Replace the two totals by the counts for the displayed ranges. Enumerating the larger supersets is harmless, so these corrections do not affect coverage.

Verifiable source: Full paper, version 1

Hensel-lift example: The inverse variable changes from d_0 to an undefined g_0

Pages 13–14 · Example $(\gamma, \alpha, \beta) = (5039, 20, 1)$ · arXiv:2607.27555v1

Typo

The construction defines d_0 by $d_0 f'(r_0) \equiv 1 \pmod{2^{100}}$, but the numerical example calls the second initial value g_0. Replace g_0 by d_0. No variable g_0 is otherwise defined, and the subsequent iteration uses the d_j, so the correction is unique and harmless.

Verifiable source: Full paper, version 1

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1
2. Tijdeman, On integers with many small prime factors
3. Ridout, Rational approximations to algebraic numbers

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.