MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On C^k-functions mapping $\mathbb{Q}$ into itself and Mahler's problem on Liouville numbers

Jean Lelis, Carlos Gustavo Moreira, Elaine Silva

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The rigidity theorem, the dense uncountable bump-function construction, and the Liouville-preservation corollary are correct. Their printed proofs contain several substantive omissions and incorrect intermediate inferences, but each has a verified repair that preserves the stated hypotheses and conclusions.

Theorem 1.1: Rigidity under the denominator bound

Pages 2 and 4–7 · Theorem 1.1 and Section 2.2 · arXiv:2607.24427v1

Correct

After cancelling the possible power of x common to the Padé numerator and denominator, the rational approximant has degree at most k and differs from the Taylor polynomial by one order more than the combined rational-denominator exponent. This still forces eventual zeros on every reciprocal sequence. The corrected divided-difference estimate then propagates equality across the connected interval, and a coprime representation excludes every genuine pole by continuity. These repairs verify the theorem with all of its stated derivative and denominator hypotheses.

Verifiable source: Full paper, version 1

Theorem 1.2: Dense uncountable family with controlled rational denominators

Pages 3 and 7–9 · Theorem 1.2 and Section 3.1 · arXiv:2607.24427v1

Correct

Order the rationals in $[0, 1)$ by nondecreasing reduced denominator and choose all translated coefficients stage by stage. Rational separation makes every later bump vanish at each value already fixed, while every earlier contribution is fixed before the current coefficient is selected. A uniform convergent-denominator tail estimate establishes C^k convergence; multiplying the normalization constant by a harmless fixed factor corrects the floor estimate. The support exclusion for $C(2)$ and the denominator upper bound are unchanged.

Verifiable source: Full paper, version 1

Corollary 1.3: Dense uncountable family preserving all Liouville numbers

Pages 3 and 10 · Corollary 1.3 and Section 3.3 · arXiv:2607.24427v1

Correct

At a rational of denominator q, write $N = C\lfloor q^t \rfloor$. The available interval of output values contains two consecutive multiples of $1/N$; at least one has reduced denominator at least $\sqrt{N}$. Selecting it forces denominators to infinity, while the independent choices at the integer stages retain uncountably many functions. Lemma 3.1 then gives $f(\mathcal{L}) \subseteq \mathcal{L}$. Finally, although the manuscript's uniqueness claim for algebraic functions is too strong, only countably many global algebraic functions can map every rational to a rational, so deleting them still leaves an uncountable family.

Verifiable source: Full paper, version 1

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The main conclusions admit verified repairs, but the printed arguments contain false intermediate assertions in the Padé step and divided-difference estimate, omit the simultaneous-support and uniform-convergence checks in the bump construction, and assert denominator growth and algebraic uniqueness without valid proofs. Two literal notation defects are reported in yellow.

Theorem 1.1, Padé denominator at the origin: The constructed denominator may vanish at zero

Pages 5–6 · Equation (3), properties (ii)–(iii), and the first eventual-zero argument · arXiv:2607.24427v1

Incorrect as written · verified repair

The homogeneous system does not ensure $Q_k(0) \neq 0$. If $r = \mathrm{ord}_0\, Q_k > 0$, then $K = S/Q_k$ need not be bounded near zero and $g = f - \widetilde{R}_k$ need not have all derivatives through order $2k$ equal to zero. For example, when $k = 1$ and $F_2(x) = x^2$, the displayed system forces Q_1 to be a multiple of x. The repair is to write $Q_k = x^r Q_0$ and, from $Q_k F_{2k} = P_k + x^{2k+1} S$, also $P_k = x^r P_0$. Then $\deg P_0, \deg Q_0 \leq k - r$, $R_k - F_{2k} = O(x^{2k+1-r})$, and $\mathrm{den}(f(p/q) - R_k(p/q)) \ll q^{2k-r}$. The extra one power of vanishing still proves $g(n/q) = 0$ for all sufficiently large q. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Theorem 1.1, divided-difference lower bound: The displayed estimate drops the factor n^{2k+1}

Page 6 · Displays following Equations (5)–(6) · arXiv:2607.24427v1

Incorrect as written · verified repair

The exact formula contains n^{2k+1} in the denominator, but the next lower bound writes $M_n^{2k+1}/M_n^{2k} = M_n$ after mentioning only $\binom{M_n+2k+1}{M_n} \geq 1$ and $M_n \gg n$. That inference does not follow from the stated lower bound. It is repaired by using

$$\binom{M_n + 2k + 1}{M_n} \geq \frac{M_n^{2k+1}}{(2k+1)!}.$$

Together with $|g(n/M_n)| \gg M_n^{-2k}$ and $M_n \gg n$, the exact formula gives $|g^{(2k+1)}(\xi_n)| \gg M_n$, which is the required contradiction. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Theorem 1.1, continuation across roots of Q_k: A root of the displayed denominator need not be a pole

Pages 6–7 · Final continuation argument · arXiv:2607.24427v1

Incorrect as written · verified repair

The proof identifies every nonzero root of Q_k with a pole of P_k/Q_k, but P_k and Q_k were not shown to be coprime, so such a root can be removable. Cancel all common factors before defining the connected component U. Its boundary points inside Ω are then genuine poles of the reduced rational function, and continuity of f excludes them exactly as intended. The reciprocal-sequence argument also applies with negative numerators if a hypothetical nonzero interval lies to the left of the origin. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Theorem 1.2, coefficient induction: The evaluation omits other translates and does not justify future noninterference

Pages 7–8 · Definition of f and the induction choosing $\lambda_{m,n}$ · arXiv:2607.24427v1

Incomplete as written · verified repair

At $x = m + p_n/q_n$, the displayed equality includes only bumps with the same translate m. A bump from another translate can be nonzero: for instance, at $x = m + 5/6$, the integer-centered bump $\psi_{m+1,0}$ is active. The formula also does not show that later choices leave the selected value unchanged. Order the enumeration by nondecreasing q_n, choose all integer coefficients first, and then proceed stagewise across m. Earlier cross-translate terms are then fixed and can be included in the base value. If a later center s has denominator $q_j \geq q_n$, rational separation gives

$$|x - s| \geq \frac{1}{q_n q_j} \geq \frac{1}{q_j^2} > \frac{1}{4q_j^2},$$

so its bump vanishes at x. Thus every rational value remains fixed and retains its denominator bound. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Theorem 1.2, C^k convergence and distance estimate: Pointwise summability is not enough, and the floor inequality has the wrong direction

Page 8 · Equations (8)–(9) and the following distance estimate · arXiv:2607.24427v1

Incorrect as written · verified repair

The paper infers $\widetilde{f} \in C^k$ from a pointwise convergent majorant without proving uniform convergence of the differentiated tails. In addition, $\lfloor q^t \rfloor \leq q^t$, so the printed replacement of $q^{2j}/\lfloor q^t \rfloor$ by $q^{-(t-2j)}$ is not valid. For $q \geq 2$, one has $\lfloor q^t \rfloor \geq q^t/2$. If the active denominator tail begins at Q, the convergent recurrence gives $q_{i+s} \geq F_{s+1} q_i$, hence the jth derivative tail is uniformly bounded by a constant times

$$Q^{2j-t} \sum_{s \geq 0} F_{s+1}^{2j-t},$$

which tends to zero for $j \leq k$ because $t > 2k$. Taking C strictly larger than $2(k+1)MA_t/\varepsilon$ then proves both termwise C^k convergence and $d(f, g) < \varepsilon$. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Corollary 1.3, denominator growth: The required lower-denominator selection is absent

Page 10 · Sentence asserting $\mathrm{den}(f(p/q)) \to \infty$ · arXiv:2607.24427v1

Incomplete as written · verified repair

Theorem 1.2 as printed proves only an upper bound and does not itself imply that image denominators tend to infinity. At the stage for denominator q, put $N = C\lfloor q^t \rfloor$. Varying the coefficient makes the output range over an interval of length $2/N$, which contains consecutive grid values a/N and $(a+1)/N$. The integers $\gcd(a, N)$ and $\gcd(a+1, N)$ are coprime and their product divides N, so at least one of these two rationals has reduced denominator at least $\sqrt{N}$. Selecting that value forces uniform denominator growth as $q \to \infty$. Uncountability is retained by the two independent choices at every integer stage, which are irrelevant to the $q \to \infty$ condition. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Corollary 1.3, algebraic exceptional functions: Local equality does not force global equality for finite-smooth algebraic branches

Page 10 · Final identity-principle argument · arXiv:2607.24427v1

Incorrect as written · verified repair

A global C^k algebraic function can switch branches at a singular point. For example, the two C^k functions obtained from the branches $\pm x^{k+1}\sqrt{1+x^2}$ can be chosen to coincide on $x > 0$ and differ on $x < 0$; both satisfy $y^2 = x^{2k+2}(1+x^2)$. Thus equality on one analytic interval does not prove that two such functions are globally identical. The corollary needs only countably many exceptions. The graph of an algebraic function is contained in a finite union of irreducible plane curves. If the function maps $\mathbb{Q}$ into $\mathbb{Q}$, every component used over a nonempty interval contains infinitely many rational graph points; these are Zariski dense in that component, so Galois invariance shows that the component is defined over $\mathbb{Q}$. There are only countably many finite unions of such rational curves. For each fixed union, the branch and pairwise-intersection abscissae form a finite set, and only finitely many global C^k branch selections are possible. Thus the algebraic members form a countable set; removing them from the constructed uncountable family still proves the corollary. Repair classification: Verified repair.

Verifiable source: Full paper, version 1

Divided-difference point count: The number of interpolation points is off by one

Page 6 · First paragraph, immediately before Equation (4) · arXiv:2607.24427v1

Typo

The list $\alpha_0, \ldots, \alpha_{2k+1}$ contains $2k+2$ distinct points, not $2k+1$. Replace the printed count by $2k+2$. Equation (4) and every subsequent product already use the complete list, so the correction is mechanical and harmless.

Verifiable source: Full paper, version 1

Definition of $\mathcal{L}_\Psi$: The approximation exponent is unquantified

Page 9 · Lemma 3.1, definition of $\mathcal{L}_\Psi$ · arXiv:2607.24427v1

Typo

The displayed set uses an exponent n without binding it. Replace the display by the sequential formulation used immediately in the proof: there exist distinct rationals p_n/q_n with $q_n \to \infty$ and

$$0 < |\xi - p_n/q_n| < (\Psi(q_n))^{-n}$$

for every $n \geq 1$. This uniquely matches the subsequent argument and leaves Lemma 3.1 unchanged.

Verifiable source: Full paper, version 1

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.