MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On a Diophantine Equation with Jacobsthal and Fibonacci Numbers

Daeyeoul Kim, Zekiye Pinar Cihan, Zeynep Demirkol Ozkaya, Ilker Inam

Paper	arXiv:2607.20763v1
Source version	Submitted 22 July, 2026; announced July 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	April 30, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

No counterexample was found to the claimed unique solution. Theorem 2.1 is not able to be verified because two parameter-dependent continued-fraction reductions are applied using single numerical values that fail for other admissible parameters. Exact enumeration confirms the theorem inside the final claimed box $m \leq 236$, but the printed argument does not validly reduce every solution to that box.

Theorem 2.1: The asserted complete classification is not established

Pages 2 and 9–11 · Theorem 2.1 and final reduction · arXiv:2607.20763v1

Not able to verify

The exact claim is that $F_k F_l F_m = J_n$ with $2 < k < l < m$ has only $(k, l, m, n) = (5, 7, 8, 12)$. The linear-form estimates give a large preliminary bound, and an exact-integer enumeration verifies the stated solution is unique whenever $m \leq 236$. However, the two Dujella–Pethő steps intended to prove $l \leq 114$ and then $m \leq 236$ do not establish positive epsilon uniformly over the still-variable indices. Concrete admissible parameter values make the displayed epsilon negative. Consequently the proof does not cover all possible solutions, and no independent proof or counterexample is supplied here.

Verifiable source: Published version of the reviewed result

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The second and third continued-fraction reductions contain unresolved uniformity gaps, and the final finite verification is absent. Two earlier numerical calculations are also incorrect as written, although local repairs preserve the preliminary bounds. Four mechanically determined notation defects are reported separately as typos.

Second and third Dujella–Pethő reductions: The quoted epsilon values are not uniform in the remaining indices

Pages 10–11 · Reductions following Equations (4.16) and (4.21) · arXiv:2607.20763v1

Incomplete as written

For the second reduction, $\mu_k = \log(5/(3F_k))/\log\alpha$ varies with k, but the paper uses one convergent q_{92} and the single value $\varepsilon = 0.00120395$ to treat every $k \leq 110$. With the printed q_{92} and $M = 8.24 \times 10^{42}$, direct high-precision reevaluation at the admissible value $k = 98$ gives $\|\mu_{98}q_{92}\| \approx 0.00513103040825$ and $M\|\tau q_{92}\| \approx 0.0153446398986$, hence $\varepsilon \approx -0.0102136094904$. Lemma 3.3 is therefore inapplicable there. Likewise, the third reduction has $\mu_{k,l} = \log(\sqrt{5}/(3F_kF_l))/\log\alpha$ but uses one value for all $k \leq 110$, $l \leq 114$; at $(k,l) = (72,109)$ its printed q_{93} and $M = 2.04107 \times 10^{43}$ give $\varepsilon \approx -0.0357465065164$. A separate positive-epsilon convergent, or another reduction, is required for every uncovered parameter choice. Repair classification: No repair supplied.

Verifiable source: Dujella–Pethő reduction lemma

End of the proof of Theorem 2.1: The finite box is not checked in the manuscript

Page 11 · Final two sentences before the acknowledgements · arXiv:2607.20763v1

Incomplete as written · verified local repair

After asserting $m \leq 236$, the proof immediately says it is finished; it never checks the finitely many triples $3 \leq k < l < m \leq 236$. An exact-integer enumeration, generating F_r and J_s from their defining recurrences and comparing every product $F_kF_lF_m$ with the Jacobsthal values for $n < 3m$, returns only $(5,7,8,12)$. This repairs the omitted finite step itself, but it does not repair the earlier failure to establish $m \leq 236$.

Verifiable source: Full paper, version 1

First Dujella–Pethő reduction: The displayed epsilon omits the subtraction term

Page 9 · Computation using q_{92} after Equation (4.23) · arXiv:2607.20763v1

Incorrect as written · verified repair

Lemma 3.3 defines $\varepsilon = \|\mu q\| - M\|\tau q\|$. For the printed q_{92} and $M = 6.13 \times 10^{43}$, the quoted 0.284374 is $\|\mu q_{92}\|$ alone; the omitted term is approximately 0.114153692450, so the actual epsilon is approximately 0.170220186049. It remains positive, and the corrected threshold is $k < 110.481$, which still yields $k \leq 110$. Repair classification: Verified repair. This local correction does not resolve the later parameter-dependent gaps.

Verifiable source: Dujella–Pethő reduction lemma

Third Matveev estimate: The product uses a smaller height bound than the one established

Page 8 · Display immediately after the choice of A_3'' · arXiv:2607.20763v1

Incorrect as written · verified local repair

The proof establishes and chooses $A_3'' = 4.25 \times 10^{24}(1 + \log(3m))^2$, but the next application of Matveev's theorem multiplies by $4.23 \times 10^{24}(1 + \log(3m))^2$. The smaller value has not been justified. Retaining 4.25 gives the valid bound $\log |\Lambda_3| > -2.808 \times 10^{36}(1 + \log(3m))^3$ and, conservatively, $m < 5.95 \times 10^{42}$ in place of 5.93×10^{42}. Repair classification: Verified local repair. The later choices of M must be adjusted accordingly, and the uniformity gap remains.

Verifiable source: Matveev's lower-bound theorem

Fibonacci characteristic polynomial: The constant term has the wrong sign

Page 3 · Paragraph following Equation (4.2) · arXiv:2607.20763v1

Typo

The polynomial is printed as $\Psi(X) = X^2 - X + 1$. Replace $+1$ by -1, giving $\Psi(X) = X^2 - X - 1$. The displayed roots $\alpha = (1 + \sqrt{5})/2$ and $\beta = (1 - \sqrt{5})/2$, and every subsequent use of $\alpha\beta = -1$, uniquely determine the correction. The argument itself consistently uses the corrected polynomial.

Verifiable source: Full paper, version 1

Statement of Matveev's theorem: The defining linear form is missing its final minus one

Page 2 · Theorem 3.1, Equation (3.5) · arXiv:2607.20763v1

Typo

The cited theorem prints $\Lambda = \alpha_1^{b_1} \cdots \alpha_t^{b_t}$, but the stated lower bound and all three later applications require $\Lambda = \alpha_1^{b_1} \cdots \alpha_t^{b_t} - 1$. Insert -1. The applications define $\Lambda_1, \Lambda_2, \Lambda_3$ in that corrected form, so this notation defect does not itself alter them.

Verifiable source: Matveev's lower-bound theorem

Height estimate for $3F_k/5$: The Fibonacci exponent uses l instead of 1

Page 6 · Last displayed estimate · arXiv:2607.20763v1

Typo

The bound is printed with $(k - l) \log \alpha$ and is followed immediately by the bound $\log(15) + k \log \alpha$. Replace $k - l$ by $k - 1$, using $F_k \leq \alpha^{k-1}$ from Equation (4.3). The intended correction is unique and the next line already uses a valid weaker consequence.

Verifiable source: Full paper, version 1

Continued-fraction threshold displays: The inequality directions contradict the invoked lemma and the conclusions

Pages 9–11 · Displays immediately before $k \leq 110$, $l \leq 114$, and $m \leq 236$ · arXiv:2607.20763v1

Typo

Lemma 3.3 says there is no solution when $w \geq \log(Aq/\varepsilon)/\log B$. Therefore an existing solution must satisfy $w < \log(Aq/\varepsilon)/\log B$. Replace the printed lower-bound signs in the three concluding displays by upper-bound signs. This is mechanically forced by the upper bounds stated on the following lines. Correcting these symbols does not repair the missing positive-epsilon checks in the second and third reductions.

Verifiable source: Dujella–Pethő reduction lemma

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 1
2. Published version of the reviewed result
3. Dujella–Pethő reduction lemma
4. Matveev's lower-bound theorem

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.