MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Bounded trajectories of quasi-rays on homogeneous spaces and Diophantine approximation with weight functions

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Paper	arXiv:2607.17021v1
Source version	Submitted 19 July, 2026; announced July 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	July 19, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The central results checked are correct under their stated hypotheses and cited inputs. The only detected defect is a mechanical index typo in the power-weight example; it is reported in yellow and does not lower the substantive status.

Theorem 1.4: Thickness for bounded trajectories of quasi-rays

Pages 3 and 11–18 · Theorem 1.4 and its proof · arXiv:2607.17021v1

Correct

The reduction to Theorem 3.1, the thinning argument for a quasi-ray, the quantitative equidistribution and tessellation estimates, and the strongly tree-like Cantor construction yield full local Hausdorff dimension in the expanding horospherical subgroup. Bounded thickening and the local product chart then give the stated thickness of $B(F)$ in G/Γ. The quantifiers over the quasi-ray and nonempty open subsets are preserved.

Verifiable source: Full paper, version 1

Theorem 3.1: Uniform Cantor estimate for sufficiently interior quasi-rays

Pages 12–18 · Theorem 3.1 · arXiv:2607.17021v1

Correct

After the distance-from-walls reduction, the cited effective equidistribution estimate is uniform for the required increments. The chosen tessellation scale leaves a positive proportion of children at every stage, their diameters shrink exponentially, and the tree-like dimension bound tends to $\dim U$ as the scale parameter grows. These estimates establish the theorem in every required open subset.

Verifiable source: Ronggang Shi, effective equidistribution input

Theorem 1.5: Weighted Diophantine application

Pages 5 and 18–23 · Theorem 1.5 and Section 4 · arXiv:2607.17021v1

Correct

Quasimultiplicativity supplies uniform positive increments, the weight functions are discretized into a quasi-ray in the super-expanding cone, and Proposition 4.4 gives the required Dani correspondence through Mahler's compactness criterion. Applying Theorem 1.4 and the local bi-Lipschitz parametrization of the unipotent subgroup gives the claimed thickness.

Verifiable source: Related weighted transference input

Power-weight example: The exponent index should agree with the function index

Page 5 · Paragraph following Theorem 1.5 · arXiv:2607.17021v1

Typo

As printed, the standard example reads $\beta_j(T) = T^{b_i}$. Replace b_i by b_j, giving $\beta_j(T) = T^{b_j}$. The function is indexed by j, the surrounding tuple is $(b_1, \ldots, b_n)$, and no later step uses the mismatched index. The correction is unique, mechanical, and harmless through every downstream use.

02 / PROOFS

Proofs

Correct

The main proofs and every material cited input checked are correct and complete. One genuine endpoint-domain mismatch has a unique verified local repair; its complete downstream scope was checked, so it changes no result and does not lower the overall proof status.

Theorem 1.4 via Theorem 3.1: Reduction, thinning, and local product argument

Pages 11–18 · Sections 2–3 · arXiv:2607.17021v1

Correct and complete

The thinning argument, bounded-thickening implication, Cantor construction, and slicing step have the required quantifiers. Proposition 2.10 supplies separation from the cone walls and Theorem 3.1 gives full local dimension in U. The set $B(F)$ is Borel: an exhaustion by compact sets writes it as a countable union of arbitrary intersections of closed preimages. The local product chart and slicing input therefore apply and transfer thickness to G/Γ.

Verifiable source: Kleinbock–Margulis bounded-orbit slicing precedent

Shi effective-equidistribution input: The cited hypotheses match the manuscript's expanding-cone regime

Pages 8–10 · Theorem 2.5, Lemma 2.6, and Corollary 2.7 · arXiv:2607.17021v1

Correct and complete

In Shi's theorem, H is the normal product of the simple factors on which U projects nontrivially; irreducibility and the absence of compact factors supply the required spectral gap; the acting element is nontrivial on every H-factor; and its expanding horospherical subgroup is U. For increments in the super-expanding cone, Lemma 2.6 identifies Shi's floor function with distance from the relevant cone wall. Corollary 2.7 therefore invokes the uniform compact-set error estimate in its stated regime.

Verifiable source: Ronggang Shi, Expanding Cone and Applications to Homogeneous Dynamics

Theorem 3.1 construction: Tessellation and strongly tree-like dimension input

Pages 12–18 · Lemma 3.3 and Propositions 3.4–3.6 · arXiv:2607.17021v1

Correct and complete

The tessellation count, return estimate, positive-child condition, boundary-null intersections, shrinking diameters, and retained-density bound meet the stated tree theorem. Proposition 3.6 supplies at least the required positive proportion of children, while Lemma 3.3 gives exponential contraction. The rendered source uses compact closures of the tessellation pieces, so the compact-set hypotheses are met even though text extraction can omit the overlines.

Verifiable source: Kleinbock–Margulis tessellation and slicing source

Observation 4.2, Equation (4.5): The endpoint $t = 0$ should be included

Page 19 · Observation 4.2, Equation (4.5); used on pages 20–22 · arXiv:2607.17021v1

Minor formal correction

Equation (4.5) is printed for $t > 0$, but the first increment in Observation 4.3 uses $t = (k-1)t_0 = 0$ when $k = 1$. Replace $t > 0$ by $t \geq 0$, or state the endpoint extension separately. The proof of (4.5) works directly at $t = 0$: the relevant endpoint values are $\alpha_i(1) = \beta_j(1) = 1$, and the quasimultiplicative inequalities allow $RT = 1$ for α_i and $T = 1$ for β_j. Repair classification: Verified repair. This fixes exactly the initial increment; every later increment is already in the printed domain.

Proposition 4.4: Generalized Dani correspondence

Pages 20–22 · Proposition 4.4 · arXiv:2607.17021v1

Correct and complete

Normalization (4.3) gives determinant one, Mahler's compactness criterion applies, and vectors with $q = 0$ have norm at least 1. Both implications correctly rescale the systems of inequalities using the lower and upper quasimultiplicative estimates. Omitting a bounded initial t-interval is harmless by compactness and continuity.

Proof of Theorem 1.5: Application to weighted approximation

Page 23 · Proof of Theorem 1.5 · arXiv:2607.17021v1

Correct and complete

Observation 4.3 produces exactly the quasi-ray required by Theorem 1.4, Proposition 4.4 identifies its bounded trajectories with the weighted badly approximable set, and the matrix-to-unipotent map is locally bi-Lipschitz. The hypotheses and parameter ranges of the three inputs match the final application.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2607.17021v1
2. Full paper, version 1
3. Ronggang Shi, Expanding Cone and Applications to Homogeneous Dynamics
4. Neckrasov, weighted inhomogeneous approximation via transference
5. Kleinbock-Margulis, Bounded Orbits of Nonquasiunipotent Flows on Homogeneous Spaces

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.