MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Entropy Density of Uniquely Ergodic Measures for Full Shifts over Amenable Residually Finite Groups

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Paper	arXiv:2607.16994v1
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Prompt	Standardized prompt
Novelty cutoff	July 18, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The approximation theorem and its density consequence are supported. Theorem 27 constructs uniquely ergodic full-shift measures approximating an arbitrary invariant measure for an amenable residually finite group, with matching finite-pattern statistics and entropy. Corollary 28 concludes that uniquely ergodic measures are entropy dense in the invariant-measure simplex in the topology specified by the paper.

Theorem 27: Approximation by uniquely ergodic full-shift measures with entropy control

Pages 17–20 · Theorem 27 and Corollary 28 · arXiv:2607.16994v1

Correct

For a countable amenable residually finite group acting on a finite-alphabet full shift, every invariant measure is approximated in the stated weak/d-bar sense by uniquely ergodic measures constructed from periodic tilings. The approximants' entropies converge to the target entropy rather than merely satisfying a one-sided inequality. The finite-index subgroup chain and Følner fundamental domains required by residual finiteness are chosen once in the construction. The result covers nonergodic target measures because the jigsaw block frequencies encode their full finite-pattern law.

Corollary 28: Entropy density of uniquely ergodic measures

Pages 19–20 · Corollary 28 · arXiv:2607.16994v1

Correct

Given an invariant measure, Theorem 27 supplies uniquely ergodic measures converging weakly and with entropy converging to its entropy. This is exactly entropy density: every neighbourhood of the measure and entropy interval contains such an approximant. The full shift and finite alphabet hypotheses used for d-bar entropy continuity are retained in the corollary.

Universally good points: Uniform pattern frequencies in the orbit closure

Pages 10–17 · intermediate propositions · arXiv:2607.16994v1

Correct

The hierarchical construction makes the frequency of each finite pattern nearly identical in every high-level macroblock. Summability of level errors gives a common limit uniformly over translates and over all configurations in the orbit closure. The standard uniform-frequency criterion then yields unique ergodicity. This stronger uniform statement is what permits the approximation to be represented by a uniquely ergodic measure rather than only by one generic configuration.

02 / PROOFS

Proofs

Correct

Universally good points and entropy continuity. Large Følner fundamental domains are tiled by labeled blocks sampled from the target finite-pattern distribution. A hierarchical jigsaw rule places every permitted macroblock with prescribed frequency inside all sufficiently large superblocks. Boundary proportions are summable, so every point in the resulting orbit closure has the same limiting pattern frequencies; this proves unique ergodicity. Coupling each jigsaw configuration with the original block labels changes symbols only near boundaries and on a chosen small exceptional set, giving the d-bar estimate. Entropy continuity in d-bar for finite alphabets then gives convergence of entropy.

Jigsaw construction: Universally good points and entropy continuity

Pages 8–20 · Sections 3–5 · arXiv:2607.16994v1

Correct and complete

Large Følner fundamental domains are tiled by labeled blocks sampled from the target finite-pattern distribution. A hierarchical jigsaw rule places every permitted macroblock with prescribed frequency inside all sufficiently large superblocks. Boundary proportions are summable, so every point in the resulting orbit closure has the same limiting pattern frequencies; this proves unique ergodicity. Coupling each jigsaw configuration with the original block labels changes symbols only near boundaries and on a chosen small exceptional set, giving the d-bar estimate. Entropy continuity in d-bar for finite alphabets then gives convergence of entropy.

Sections 3–4: Residual tilings and jigsaw frequency control

Pages 8–17 · Sections 3–4 · arXiv:2607.16994v1

Correct and complete

Finite-index subgroups provide periodic fundamental-domain tilings, and amenability selects domains with vanishing boundary for every fixed finite test set. Target blocks are distributed among supertiles using rational approximations to their probabilities. The rounding error and tile boundary error are made summable over levels. Induction then proves uniform convergence of all cylinder frequencies and minimal recurrence of the constructed orbit closure.

Section 5: d-bar coupling and entropy convergence

Pages 17–20 · Section 5 · arXiv:2607.16994v1

Correct and complete

A coupling pairs each jigsaw supertile with the target block used to label its interior. Disagreement is confined to boundaries, rounding corrections, and the chosen atypical blocks, whose combined upper density tends to zero. Hence the d-bar distance tends to zero. For finite-alphabet amenable shifts, the cited entropy-continuity inequality bounds the entropy difference by a function of that distance, yielding the two-sided convergence required in Theorem 27.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2607.16994v1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\mathrm{delta})$ is proved for every sufficiently small $\mathrm{delta} > 0$ and P is immediately monotone in the required direction, then the claim that $P(\mathrm{delta})$ holds for every $\mathrm{delta} > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.