

Correctness, proof completeness, and novelty audit

Optimal rates of uniform convergence for weighted Birkhoff averages via almost all rotations

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

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01 / STATEMENTS

Statements

Contains wrong statements

The lower-bound existence assertions are false in part of their stated parameter range because Definition 2.3 makes the claimed observable class empty whenever its global Fourier-envelope sum diverges. The paper's explicit sparse Fourier constructions do converge absolutely, but that does not place them in the printed class. The upper-rate assertions of Theorem 3.2 are correct.

Theorem 3.1: The claimed observable need not belong to the class in the theorem

Pages 8 and 12–13 · Definition 2.3 and Theorem 3.1 · arXiv:2607.06907v2

Incorrect

Definition 2.3 includes the class condition

$$\sum_{k \in \mathbb{Z}^d \setminus \{0\}} \tilde{\Delta}(\|k\|_{\ell^\infty})^{-1} < \infty.$$

Theorem 3.1 assumes only $x^\varepsilon \leq \tilde{\Delta}(x) \leq \exp(x^\zeta)$ for some $\varepsilon > 0$ and $0 < \zeta < \beta^{-1}$. Take $\tilde{\Delta}(x) = x^{1/2}$, which satisfies those hypotheses, but in the discrete one-dimensional assertion the defining sum is $2 \sum_{n \geq 1} n^{-1/2} = \infty$. Consequently the printed class $\mathcal{R}_{\tilde{\Delta}}^L(\mathbb{T})$ is empty, contradicting the theorem's assertion that an observable in it exists. The same example also violates the two-dimensional sum in the continuous assertion and every Case (I)–(III).

Verifiable source: [Full paper, version 2](#)

Corollary 3.1: The higher-dimensional existence conclusion has the same counterexample

Page 13 · Corollary 3.1 · arXiv:2607.06907v2

Incorrect

The corollary retains the unrestricted hypothesis $\varepsilon > 0$ while asserting existence in $\mathcal{R}_{\tilde{\Delta}}^L(\mathbb{T}^d)$. For $\tilde{\Delta}(x) = x^{1/2}$, Definition 2.3 again requires a divergent lattice sum. More generally, for $\tilde{\Delta}(x) = x^L$, that sum diverges whenever $L \leq d$. Thus the asserted class member does not exist throughout a nonempty part of the corollary's stated range.

Verifiable source: [Full paper, version 2](#)

Theorem 3.2, upper bounds: The universal upper-rate assertions

Pages 14–16 and 29–42 · Theorem 3.2 and its proof · arXiv:2607.06907v2

Correct

The finite-regularity upper regimes impose enough decay for the full Fourier lattice sum used in the proof, and the C^∞ , logarithmic C^∞ , and Gevrey envelopes do so as well. The Fourier and Poisson-summation estimates, the one-dimensional Denjoy–Koksma and Abel argument, the continuous small-divisor packing estimate, and the low/high-frequency decompositions give the displayed rates uniformly in the initial point.

Theorem 3.2, finite-regularity lower bounds: The advertised all-regularity optimality range includes empty classes

Pages 14–15 and 28–29 · Case (I) and opening of the proof of Theorem 3.2 · arXiv:2607.06907v2

Incorrect

The finite-regularity clauses claim counterexamples for all polynomial exponents $L > 0$ and explicitly derive them from Theorem 3.1 and Corollary 3.1. Under the printed Definition 2.3, however, $\mathcal{R}_{x_L}^l(\mathbb{T}^d)$ is empty when $L \leq d$, since $\sum_{k \neq 0} \|k\|_{\ell^\infty}^{-L}$ diverges. The existence-based statement that the listed rate is attained therefore fails in those regimes. This finding does not affect the separately proved upper bounds or the lower clauses whose envelope satisfies the defining global summability condition.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The lower-bound construction checks absolute convergence only along its sparse frequency subsequence and incorrectly treats that as satisfying Definition 2.3’s full-lattice condition. Restricting the hypotheses to globally summable envelopes, or explicitly stating the result for the constructed sparse-series class, repairs the affected lower bounds. The upper-bound proof chain is correct and complete.

Proof of Theorem 3.1 and Corollary 3.1: Sparse absolute convergence does not verify membership in the printed class

Pages 21–29 · Sections 5.1–5.2, especially the constructed Fourier series · arXiv:2607.06907v2

Incorrect as written

The construction uses only frequencies q_{m_j} and the exponential growth of this subsequence, so $\sum_j \tilde{\Delta}(q_{m_j})^{-1} < \infty$ follows from $\tilde{\Delta}(x) \geq x^\varepsilon$. That proves the explicitly constructed sparse series is continuous and absolutely convergent. Definition 2.3 instead requires the envelope sum over every nonzero $k \in \mathbb{Z}^d$, which the argument never proves and which is false for the concrete admissible choice $\tilde{\Delta}(x) = x^{1/2}$. Downstream dependency: every membership assertion in Theorem 3.1 and Corollary 3.1, and the lower-bound clauses of Theorem 3.2, use this step. Repair classification: Verified repair after changing the affected theorem scope. Either add the full-lattice summability condition to the hypotheses, or replace membership in $\mathcal{R}_{\Delta}^L$ by membership in the explicitly defined absolutely convergent sparse-series class with the stated coefficient envelope; all subsequent lower-bound calculations then apply unchanged.

Verifiable source: [Full paper, version 2](#)

Proof of Theorem 3.2, lower bounds: The proof imports the defective class-membership conclusion

Pages 28–29 · opening paragraphs of Section 5.3 · arXiv:2607.06907v2

Incorrect as written

The proof says that all counterexample-type estimates follow directly from Theorem 3.1 and Corollary 3.1 and that convergence over the constructed subsequence justifies the weaker parameter restrictions. Subsequence convergence justifies the existence of those particular Fourier series, but it does not satisfy Definition 2.3’s independent full-lattice envelope condition. Downstream dependency: the claimed finite-regularity optimality statements for all $L > 0$. Repair classification: Verified repair only with the same restriction or class redefinition described above.

Proof of Theorem 3.2, upper bounds: Fourier, discrepancy, and frequency-splitting estimates

Pages 29–42 · Section 5.3 · arXiv:2607.06907v2

Correct and complete

In every upper-bound regime, the proof first works in a range where the Fourier expansion is absolutely summable. The discrete estimates control the weighted exponential sums, the one-dimensional refinement uses Denjoy–Koksma followed by Abel summation, and the continuous argument separates the rare small divisors before summing the remaining modes. Repeated integration by parts and the stated endpoint-flatness of the weighting functions justify the logarithmic and Gevrey low/high-frequency splits. No missing substantive case was found.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. [Full paper, version 2](#)

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be *Correct* in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an *Incorrect* theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.