MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Conditional entropy realization and approximation by uniquely ergodic measures

Xiaobo Hou, Wanshan Lin, Xueting Tian, Yi Yuan, Xutong Zhao

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Prompt	Standardized prompt
Novelty cutoff	June 27, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Correct
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The exact entropy-and-average realization theorem, its weak-star density conclusion, the three multi-horseshoe inputs, and the stated conditional-variational and Lyapunov-spectrum consequences are correct under the hypotheses given. The separate arguments for non-invertible, non-compact, and non-uniform systems supply the properties used by the common nested construction.

Theorem A: Exact constrained realization and density hold in all three classes

Pages 6–8 and 18–55 · Theorem A via Theorems 5.1, 7.4, and 9.2 · arXiv:2606.28771v2

Correct

Fix an invariant measure of entropy h and average a, with h strictly below the entropy supremum in the a-fiber. A nearby exact-a measure of entropy above h and a nearby zero-entropy exact-a measure give the initial entropy interpolation; measures with averages on the two sides of a give exact average control. At every later stage, two nested subhorseshoes retain averages on opposite sides of a, have entropy intervals shrinking to h, and have invariant-measure spaces of diameter tending to zero. Their intersection therefore supports one measure ν with $h_\nu(f) = h$ and $\int \phi \, d\nu = a$. Taking $\mathrm{supp}(\nu)$ makes the subsystem minimal and uniquely ergodic, and the first-stage distance estimates make ν arbitrarily close to the prescribed measure. The degenerate case $L_\phi = \{a\}$ is the same construction with the automatic average constraint omitted.

Theorems B, C, and D: The multi-horseshoe statements provide the claimed entropy and measure control

Pages 13–17, 31–38, and 45–53 · Theorems B, C, and D · arXiv:2606.28771v2

Correct

For a topologically expanding map, periodic separated orbit segments supplied by uniform separation are concatenated as pseudo-orbits and uniquely shadowed, producing finite full shifts for an iterate. For a transitive countable Markov shift, finite families of periodic words with controlled Birkhoff averages concatenate freely and stay in a finite alphabet, hence give compact full shifts. For a finite-alphabet subshift with non-uniform structure, edit approachability transfers sufficiently many typical words into a freely concatenable collection; the marker word of minimal period separates phases and the entropy loss is explicitly bounded. In each case the block empirical measures give the two Hausdorff estimates required by the later induction.

Theorems 2.6 and 2.8: The conditional entropy and Lyapunov-spectrum consequences follow

Pages 4–6 and 24–27 · Theorems 2.6 and 2.8 · arXiv:2606.28771v2

Correct

For Theorem 2.6, generic points give the lower entropy bound, the saturated-set estimate gives the matching upper bound over the average fiber, and Theorem A realizes every entropy strictly below that supremum by a compactly supported uniquely ergodic measure, so the suprema agree. For Theorem 2.8, multi-average conformality identifies each repeated Lyapunov exponent with the integral of the continuous normalized logarithmic Jacobian on the corresponding invariant bundle. The finite-vector version of the nested construction realizes every vector in the relative interior by a uniquely ergodic measure, which yields both displayed relative-interior and ordinary-interior equalities.

02 / PROOFS

Proofs

Correct

The central proofs are correct and complete. The shadowing construction for expanding maps, the finite-alphabet reduction for countable Markov shifts, the edit-and-marker construction for non-uniform subshifts, and the shared decreasing-horseshoe argument collectively establish every substantive conclusion.

Theorem 5.1: The nested-horseshoe proof closes entropy, average, and uniqueness simultaneously

Pages 18–24 · Lemma 5.2 and proof of Theorem 5.1 · arXiv:2606.28771v2

Correct and complete

Lemma 5.2 correctly transfers invariant measures between an f^N-invariant base and its N cyclic images; entropy is preserved for f^N and therefore scales by $1/N$ for f. At stage k, variational interpolation in the two sub-full-shifts supplies measures with entropy just above h and averages on opposite sides of a. The multi-horseshoe estimate makes $\operatorname{diam} \mathcal{M}(f, \Lambda_k) \to 0$, while topological entropies decrease to h. Upper semicontinuity gives the limiting measure entropy at least h, and the topological upper bound gives at most h. The intersection thus has exactly one invariant measure and the required entropy and average.

Theorems B and C: The two uniform symbolic models have the asserted quantitative controls

Pages 13–17 and 31–38 · proofs of Theorems B and C · arXiv:2606.28771v2

Correct and complete

In Theorem B, the shadowing accuracy is chosen below both the positive expansivity constant and the separation scale. Consequently each infinite concatenation has a unique tracer, periodic concatenations give actual periodic points, and the coding maps are conjugacies. In Theorem C, the connector lengths are negligible compared with the typical blocks; all constructed words begin in the same cylinder and can be concatenated freely. The $o(n)$ cylinder variation of every bounded uniformly continuous test function controls all invariant measures on the resulting compact shifts. The entropy cardinality estimates and the recurring return to the chosen cylinder prove the remaining clauses.

Theorems D and 9.2: The non-uniform symbolic construction and its reduction are complete

Pages 43–55 · Theorem 8.9, Lemma 8.11, and proofs of Theorems D and 9.2 · arXiv:2606.28771v2

Correct and complete

The cited symbolic approximation theorem supplies entropy-approaching ergodic measures on transitive sofic shifts, while edit approachability and the combinatorial edit bound retain exponentially many distinct free-concatenation words. The count of short-period words guarantees a marker of exact period T; deleting its cyclic T-blocks makes the MT phases disjoint in the two-sided case and gives bounded-gap specification in the one-sided case. Equations (9.29)–(9.32) control all block empirical measures and propagate the initial positive-entropy perturbation back to the original convex hull. Remark 9.1 then places the subsequent nested construction inside compact finite full shifts, where entropy upper semicontinuity is available.

Verifiable source: Climenhaga–Thompson–Yamamoto, non-uniform symbolic horseshoes

Countable-state input: The entropy-approachability input is used within its scope

Pages 29–39 · Lemmas 6.3–6.6 and proof of Theorem 7.4 · arXiv:2606.28771v2

Correct and complete

Takahasi's theorem applies to every invariant probability measure on a transitive countable Markov shift and supplies ergodic compact-support approximants with convergent entropy. The proof uses it first to replace a possibly infinite-entropy high-entropy fiber measure by finite-entropy compact approximants, and then corrects their averages with periodic measures; the correction coefficient tends to zero. After the first application of Theorem C, all later stages lie in finite full shifts, so the compact-case entropy argument applies without assuming upper semicontinuity on the original non-compact shift.

Verifiable source: Takahasi, entropy-approachability for transitive countable Markov shifts

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2606.28771v2
2. Climenhaga–Thompson–Yamamoto, non-uniform symbolic horseshoes
3. Takahasi, entropy-approachability for transitive countable Markov shifts

arXiv acknowledgement

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.