MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Transcendence and measures via the refined Diophantine exponent

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Prompt	Standardized prompt
Novelty cutoff	May 28, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The transcendence criterion and the quantitative results in Theorems A–C are correct after the harmless notation repairs recorded below. Theorem D and the stronger auxiliary criterion Theorem 6.5 are not verified: their proof uses a lower-bound lemma whose printed hypotheses are strictly broader than those of the cited source and under which the lemma is false. No counterexample to Theorem D itself was found.

Theorem A: Refined-Diophantine-exponent transcendence criterion

Pages 4 and 18–23 · Theorem A and Section 3.2 · arXiv:2605.30606v2

Correct

The sparse-mismatch data give the required Subspace-Theorem inequality. The stabilized linear relation is then separated into its widely spaced polynomial blocks by Lenstra's lacunary-factor lemma, and the limiting relation forces the sum either to lie in K or to be transcendental. The two evaluation-point and linear-form subscripts corrected below are uniquely determined notation errors and do not change the argument.

Theorem B and Corollary B.1: Quantitative transcendence measure and Mahler classification

Pages 5–6 and 23–34 · Theorem B, Corollary B.1, and Section 4 · arXiv:2605.30606v2

Correct

The quantitative Subspace Theorem, Siegel-lemma construction, Liouville lower bounds, and height estimates combine with the bounded-density repetition hypothesis to give the displayed exponent

$$(2d)^{c(\log 4d)(\log\log 4d)}.$$

The count of exceptional subspaces and the number of relevant points in each subspace have the required logarithmic sizes. The Mahler-classification trichotomy follows from these bounds and the standard height interpretation of ω_d.

Theorem C: Lacunary-series dichotomy and quantitative bounds

Pages 6 and 34–38 · Theorem C and Section 5 · arXiv:2605.30606v2

Correct

When the quotient of successive support indices is unbounded, truncations give arbitrarily strong approximations of bounded degree. In the bounded-quotient case, the modified quantitative argument and two applications of the lacunary-factor lemma exclude every U_d class and yield $\omega_d(\xi) \leq (2d)^{c \log\log(4d)}$. Reading the proof with the full coefficient sequence, as specified in the typo correction below, makes the displayed decompositions consistent.

Theorem 6.5 and Theorem D: Exclusion of U_1 and U_2 for periodic-prefix approximants

Pages 7 and 38–41 · Theorems 6.5 and D · arXiv:2605.30606v2

Not able to verify

The intended height-growth argument depends on Lemma 6.4 to give a positive lower bound for the distance from the coefficient series to each eventually periodic approximation. Lemma 6.4 is false for the arbitrary digit words permitted in its printed statement; the cited Adamczewski–Cassaigne result instead assumes the greedy β-expansion. The k-bonacci and Sturmian papers cited here verify transcendence, but they do not verify the stronger S- or T-number conclusion. No independent proof of that conclusion under the stated hypotheses, and no counterexample to it, was established in this audit.

Verifiable source: Adamczewski–Cassaigne, Lemma 8.2

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proofs of Theorems A–C are complete after local notation and rounding corrections. The proof of Theorem 6.5, and hence the stated proof of Theorem D's S/T classification, relies on a false generalization of a cited separation lemma. Its height argument also assumes consecutive approximants are distinct without proof. In the Sturmian case, ordinary equidistribution does not supply the uniform shrinking-interval estimate asserted in the text, although bounded type gives a standard repair to that latter step.

Lemma 6.4: The cited separation bound is false for arbitrary digit words

Page 38 · Lemma 6.4 · arXiv:2605.30606v2

Incorrect as written

The paper removes the greedy-expansion hypothesis from Adamczewski–Cassaigne's Lemma 8.2. Under the enlarged hypotheses, cancellation can make the two represented numbers equal. Take $\beta = 2$, $U = 1$, $V = 01$, so $b = UV^\infty = 101010\ldots$, and take $j = 3$. Let $a_i = b_i$ except that $a_2 = 0$ and $a_3 = 2$. Then $a_0 = b_0$, $a_1 = b_1$, and $a_2 \neq b_2$, exactly as required, but

$$\sum_{i\geq 0}(a_i - b_i)2^{-i} = -2^{-2} + 2 \cdot 2^{-3} = 0.$$

Thus $\xi = \alpha$, contradicting the claimed strictly positive lower bound. The cited source assumes that the digits of ξ form its greedy β-expansion and uses the corresponding greedy-tail inequality. No replacement separation theorem for the arbitrary words used in Theorem 6.5 is supplied.

Verifiable source: Adamczewski–Cassaigne, original Lemma 8.2

Claim 6.6: The height lower bound assumes two approximants are distinct

Pages 38–39 · proof of Claim 6.6 · arXiv:2605.30606v2

Incomplete as written

The proof applies Liouville's inequality to α_n and α_{n-1} without showing that $\alpha_n \neq \alpha_{n-1}$. Equality of two different eventually periodic digit words is possible in a non-integer base, and is also possible with the redundant digit alphabet allowed when β is an integer. Convergence to the assumed transcendental ξ shows only that infinitely many distinct values occur, not that consecutive ones differ. With a valid lower separation bound one could select first distinct successors and control the skipped range, but that repair presently depends on replacing the false Lemma 6.4.

Verifiable source: Full paper, version 2

Theorem D · Sturmian case: Ordinary equidistribution is not uniform for the shrinking intervals used

Pages 40–41 · proof of Theorem D for Sturmian words · arXiv:2605.30606v2

Incomplete as written · verified repair of this step

For each n, the mismatch set is the union of two intervals of length $\|u_n\theta\|$, and the proof asserts from equidistribution alone that its number of visits up to the varying endpoint v_n is $\asymp v_n\|u_n\theta\|$. Equidistribution gives an asymptotic for each fixed interval; it does not by itself give a uniform estimate for intervals shrinking with n and endpoints depending on n. This particular step can be repaired using the bounded-partial-quotient hypothesis: rotations of bounded type have return gaps $O(1/|I|)$ for every interval I, by the continued-fraction net/three-gap estimate. Since v_n is maximal before the fixed mismatch threshold is exceeded, this gives $v_n = O(1/\|u_n\theta\|) = O(u_n)$, which is the only conclusion subsequently used. This repair does not resolve the separate Lemma 6.4 defect.

Lemma 2.20: The chosen endpoint and mismatch bound require rounding and slack

Pages 15–16 · final paragraph of Lemma 2.20 · arXiv:2605.30606v2

Minor formal correction

The proof sets $t_n = \lfloor \rho q_n \rfloor - 1$ and $\delta = \rho/\nu$. The latter need not be an integer, while the former need not satisfy $t_n \geq \rho s_n$ when $s_n = q_n - 1$; for $1 < \rho < 2$ this fails whenever $\{\rho q_n\} > \rho - 1$. Fix the target ρ, run the counting estimate with any $\rho_0 > \rho$, and set $t_n = \lfloor \rho_0 q_n \rfloor - 1$ and an integer $\delta > \rho_0/\nu$. For all sufficiently large n, $t_n \geq \rho s_n$, and the mismatch count is bounded by δ. This is a local parameter repair and leaves Proposition 2.17 unchanged.

Theorem A and Claim 4.16: The special linear form and auxiliary polynomials are evaluated at the wrong symbol

Pages 20, 22, and 30 · proof of Theorem A and Claim 4.16 · arXiv:2605.30606v2

Typo

On page 20 the exceptional form must be $L_{3,v}$, not $L_{3,w}$, because the next bullet makes every $(i, w) \neq (3, v)$ a coordinate form and Equation (3.4) uses the exceptional form only at v. On pages 22 and 30, the displayed polynomials have positive powers and satisfy

$$P_n(\beta^{-1}) = \beta^{-s_n} \text{ (displayed linear relation)} = 0,$$

not $P_n(\beta) = 0$. Replace every evaluation at β in these lacunary-polynomial steps by β^{-1}. Height and root-of-unity properties are unchanged, so the degree-gap arguments remain valid.

Theorem C: Sparse coefficients are used as though they formed a full sequence

Pages 34–38 · Definition 5.1 and proof of Theorem C · arXiv:2605.30606v2

Typo

The theorem denotes by a_i the nonzero coefficient attached to the sparse exponent u_i, but its proof later writes sums over every exponent with the same symbol. Define the full sequence $\widetilde{a}_m = a_i$ when $m = u_i$ and $\widetilde{a}_m = 0$ otherwise, and read those later sums with $\widetilde{a}_m$. This makes the block decompositions and all displayed identities exact. The reference in Lemma 5.4 to the polynomial in Equation (3.7) should likewise point to Equation (4.9).

Theorem 6.5: The periodic comparison index omits the preperiod offset

Page 38 · first paragraph of the proof of Theorem 6.5 · arXiv:2605.30606v2

Typo

For $U_n = a[0, r_n]$ and $V_n = a[r_n + 1, s_n]$, the digit of $U_n V_n^\infty$ at an index $m > r_n$ is

$$a_{r_n+1+((m-r_n-1) \bmod (s_n-r_n))}.$$

The printed definition compares a_{j-1} with $a_{(j-1) \bmod (s_n-r_n)}$, omitting this offset. Insert the displayed shifted index in the definition of j_n. It is the unique comparison consistent with $U_n V_n^\infty$ and restores the intended first-mismatch bounds, but it does not repair Lemma 6.4.

Theorem D · degree-one boundary and Remark 6.7: The integer-base case and the alternative Schmidt citation need correction

Pages 40–41 · two proofs of Theorem D and Remark 6.7 · arXiv:2605.30606v2

Minor formal correction

In the k-bonacci case the proof writes $[\mathbb{Q}(\beta):\mathbb{Q}]=2$, although Theorem D allows an integer Pisot base of degree 1; replace equality by ≤ 2. Remark 6.7 cites W. M. Schmidt's paper on Northcott's theorem for a result actually due to K. Schmidt on eventually periodic greedy β-expansions. More importantly, that theorem concerns the greedy expansion, whereas the coefficient words used here are not shown to be greedy-admissible. The remark therefore does not provide the claimed alternative proof without an additional admissibility or uniqueness argument. The main proof already obtains transcendence from the correctly applicable k-bonacci and Sturmian sources.

Verifiable source: K. Schmidt, On Periodic Expansions of Pisot Numbers and Salem Numbers

Remaining proof chain: Subspace-Theorem and quantitative arguments for Theorems A–C

Pages 18–38 · Sections 3–5 · arXiv:2605.30606v2

Correct and complete

The normalized-place estimates, product-formula cancellation, quantitative Subspace-Theorem reduction, Siegel-lemma construction, Liouville inequalities, and Lenstra block separation are correctly matched to their hypotheses. After the explicit local notation corrections above, the dependencies proving Theorems A–C contain no further unsupported nontrivial step found in this audit.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 2
2. Adamczewski–Cassaigne, Lemma 8.2
3. Adamczewski–Cassaigne, original Lemma 8.2
4. K. Schmidt, On Periodic Expansions of Pisot Numbers and Salem Numbers

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.