

Correctness, proof completeness, novelty, and source audit

Subgraphs versus Orientations: Infinite families of equidistributions

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Paper	arXiv:2605.16028v1
Source version	Submitted 15 May, 2026; announced May 2026.
Audit date	August 31, 2026
Prompt	Standardized prompt
Novelty cutoff	May 15, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Sources correct and complete

This independent report links to the source paper on arXiv; it does not reproduce or redistribute that paper.

01 / STATEMENTS

Statements

Contains wrong statements

The solid-set invariance in Theorem 3.1 and its enumerative consequences through Corollary 3.7 are correct. The equivalence-class invariance in Theorem 4.5 is false under the paper's literal Section 2 arc-digraph definitions: a 0-way edge supplies no arc and therefore cannot belong to the cocycle that Definition 4.4 requires, while using both opposite arcs of one 2-way edge also conflicts with the intended underlying-edge cycle convention. A standard potential-cycle and potential-cut definition repairs the theorem, and exhaustive tests then pass. Example 3.4 and one introductory specialization also omit necessary qualifications.

Theorem 3.1 and Corollary 3.2: The subgraph-orientation invariance is correct

PDF pages 4 and 6-8 · theorem, local four-state identity, and corollary · arXiv:2605.16028v1

Correct

For a fixed edge, the two 1-way configurations and the two solid configurations have equal valid-state totals. Lemmas 3.8-3.9 correctly separate monotonicity from the cyclic-versus-acyclic upgrade, and the two claims cover totals zero, one, and two. Exhaustive enumeration checked every simple graph on at most three vertices and every pair of constraint sets, for 32,801 theorem instances; 68 selected loop and parallel-edge instances also passed.

Corollaries 3.6 and 3.7: The polynomial and generating-function identities are correct

PDF pages 5-6 · Equations (1)-(4) and their proofs · arXiv:2605.16028v1

Correct, with a proof repair recorded below

Direct polynomial expansion agrees on both sides of Equation (1) for every simple graph on at most three vertices. Equation (4) follows because a tournament is an ordered sequence of its strong components, whereas a totally cyclic fourientation is a labelled set of its strong components. The resulting relation is equivalent to the Archer-Gessel-Graves-Liang formula.

Section 2, Definition 4.4, and Theorem 4.5: The fourientation cycle and cocycle definitions do not support the stated theorem

PDF page 3 · directed cycles and cocycles of a fourientation; PDF pages 9-10 · Definition 4.4 and Theorem 4.5 · arXiv:2605.16028v1

False under the stated definitions; coherent after an explicit repair

Section 2 defines cycles and cocycles of a fourientation as those of its arc digraph. A 0-way edge has no arc, so a cocycle made entirely of 0-way edges is impossible literally. For a graph consisting of one nonloop edge with empty constraint sets, the two orientations form one cocycle-reversal class when the solid set is empty, but the 0-way and 2-way states give two surviving singleton classes when the edge is solid. Thus Theorem 4.5(ii) has counts one and two. The cycle language has the parallel ambiguity that the two opposite arcs of one 2-way edge can form a digraph 2-cycle even though the later forest interpretation must use each original edge at most once. The unique natural repair is to define potential cycles and cuts on underlying edge supports: allow a 2-way edge on a cycle without reusing it, and allow 0-way crossing edges on a cut while forbidding 2-way crossings. Under this repaired semantics, all 98,403 exhaustive cycle, cocycle, and cycle-cocycle class instances and the selected multigraph cases passed.

Example 3.4: The existential characterization needs both vertex sets nonempty

PDF page 4 · Example 3.4 · arXiv:2605.16028v1

Wrong at an omitted boundary case

If X is empty and $Y = V$, or conversely, then $B = X \times Y$ is empty and every orientation or subgraph is valid, while the displayed existential reachability and component conditions are impossible for nonempty V . Adding $X \neq \emptyset$ and $Y \neq \emptyset$ repairs the example. With that assumption, the characterization is correct: paths can be spliced through the added arcs from X to Y , and first-arc and last-arc truncation proves the converse.

Disconnected graphs in the introduction: Connected subgraphs and spanning trees require a connectivity qualification

PDF page 3 · final paragraph of the introduction · arXiv:2605.16028v1

Wrong as stated for disconnected graphs

With empty constraint sets, the introduction says cocycle classes and cycle-cocycle classes correspond to connected subgraphs and spanning trees. For two isolated vertices there is one orientation class but neither a connected spanning subgraph nor a spanning tree. Corollary 4.6 later gives the correct formulation through component-preserving connectedness. The introductory sentence should either assume that G is connected or say component-preserving spanning subgraphs and spanning forests.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

Theorem 3.1 has a sound self-contained proof after two forced notation repairs. The proof of Corollary 3.6 makes a false coefficient identification when a monomial contains both oriented arcs of one original edge, although the identity is correct and the omitted zero-coefficient case repairs it. Section 4 proves the intended potential-cycle and potential-cut theorem rather than the literal definitions stated in Section 2; after inserting those definitions, its contraction, reversal, outdegree, flow, and component-counting arguments are coherent, with several compressed analogies and local cross-reference defects still requiring repair.

Proof of Theorem 3.1: The four-state argument closes

PDF pages 6-8 · Lemmas 3.8-3.9 and Claims 1-2 · arXiv:2605.16028v1

Correct after two local notation repairs

Adding arcs preserves every cyclic constraint and can only destroy an acyclic constraint. If a chosen orientation of the active edge is cyclic, upgrading it to 2-way preserves strong components; if it is acyclic, deleting it cannot destroy a cycle witnessing a constraint from B . The two path-splicing claims then force equality of the two configuration sums. The opening must read $S \subseteq E$, not $S \subseteq V$, and the fixed object on page 6 is a fourorientation of $G \setminus e$, not necessarily an orientation.

Proof of Corollary 3.6: The printed coefficient identification omits conflicting arc variables

PDF page 5 · proof of Equation (1) · arXiv:2605.16028v1

Incorrect as written; uniquely repairable

Take one edge joining vertices zero and one, let A be empty, and put both opposite arcs in B . The coefficient of $z_{01}z_{10}$ is zero on each side of Equation (1), because one original edge cannot be two different 1-way arcs. The claimed valid-subgraph coefficient after deleting that edge is one, since the two adjoined arcs form a directed cycle. The proof is repaired by first observing that every monomial using both arcs of one edge has coefficient zero on both sides, and applying the deletion argument only when the chosen arcs have distinct underlying edges. The displayed polynomial identity itself passed independent exhaustive expansion.

Theorem 4.5: The proof needs the missing potential-cycle and potential-cut definitions

PDF pages 8-19 · Lemma 4.2 through Lemma 4.19 and the three class counts · arXiv:2605.16028v1

Incomplete for the stated theorem; coherent for the repaired theorem

The proof repeatedly treats 2-way edges as usable but fixed on an underlying cycle, and 0-way edges as usable but fixed on an underlying cut. Those are standard potential-cycle and potential-cut semantics, not the arc-digraph definition printed in Section 2. Once the missing definitions are supplied, Lemma 4.9 gives disjoint reversal supports, Lemmas 4.11, 4.14, and 4.17 bound adjacency degrees, the outdegree and cycle-flow potentials rule out monochromatic nontrivial cycles, and the block associations give the required componentwise counts. Independent enumeration found no counterexample under that repaired semantics, but computation is not a proof and cannot repair the missing definition silently.

Fact 4.10 and Lemma 4.9: The disjoint decomposition input is valid

PDF pages 11-12 · Fact 4.10 and proof of Lemma 4.9 · arXiv:2605.16028v1

Correct after two symbol repairs

The symmetric difference vector of two directed cycles or cocycles is a signed zero-one vector in the corresponding cycle or cocycle lattice, hence decomposes conformally into edge-disjoint directed cycles or cocycles. The cited Backman, Backman-Baker-Yuen, and Ding sources contain the needed graph or regular-matroid statements. In the focal proof the reversal sequence is introduced as $C_1, \dots, C_n$ but once called $C_1, \dots, C_k$, and the reduced orientation α' is printed as coming from undefined ψ rather than ϕ' . Both corrections are forced by the surrounding sentence.

Section 4 exposition: Several cross-references and analogous-proof transitions need correction

PDF pages 15-19 · cocycle and cycle-cocycle cases · arXiv:2605.16028v1

Repairable exposition gaps

The cocycle adaptation should say that Corollary 4.15 replaces Corollary 4.12 and Lemma 4.16 replaces Lemma 4.13; the printed sentence reverses both roles. The proof of Corollary 4.18 must use $\mathcal{G}_S''(A, B)$, not $\mathcal{G}_S(A, B)$, and the final component count must again use the double-prime graph, not $\mathcal{G}_S'(A, B)$. Lemmas 4.14, 4.16, 4.17, and 4.19 are justified only by analogy. Their intended arguments do transfer after the definition repair, but writing the lifted block-association argument explicitly would close the proof at publication standard.

Local typesetting defects: Three further printed slips have forced readings

PDF pages 4, 5, and 15 · Remark 3.5, proof of Corollary 3.6, and subsection 4.3 heading · arXiv:2605.16028v1

Typos

Remark 3.5 writes the two members of B with unordered-pair braces even though $B \subseteq V^2$; they must be the ordered pairs $(1, 2)$ and $(2, 3)$. The proof of Corollary 3.6 cites 'Theorem 1' through the equation label, but must cite Theorem 3.1. The subsection heading says 'Proof of of'. None of these three slips changes a mathematical conclusion.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

04 / SOURCES

Sources

Sources correct and complete

The exact official arXiv v1 PDF and source archive are frozen, and every proof-critical input and every historical attribution evaluated in this audit was checked in an exact arXiv version, an official author-hosted journal copy, or the authors' official book copy. The citations attached to Fact 4.10 support the needed disjoint cycle and cocycle decomposition; Archer-Gessel-Graves-Liang and Flajolet-Sedgewick support Corollary 3.7; and Gioan's author-hosted published article supports the historical reversal-class attributions and the rooted-connectivity specialization. Remaining citations supply standard background or contextual attribution rather than an unproved logical step used by the focal arguments audited here.

Review form: The exact official v1 record is frozen

Official abstract, 20-page PDF, and source archive · arXiv:2605.16028v1

Exact source verified

The official record was submitted on May 15, 2026. The PDF and TeX title use Oliver Bernardi and Jonathan J. Fang; the submission-history contact is displayed as Olivier Bernardi. The license is the arXiv nonexclusive distribution license. The report treats the PDF name as authoritative for this paper and does not infer a separate author identity from the contact spelling.

Verifiable source: [Official arXiv:2605.16028v1](#)

Fact 4.10: The cycle and cocycle decomposition citations support the imported fact

PDF pages 11-12 · Fact 4.10 and Lemma 4.9 · exact arXiv fallbacks 1408.3962v3, 1701.01051v3, and 2109.01930v4 · arXiv:2605.16028v1

Applicable and sufficient

Backman proves that differences of cut-reversal-related partial orientations decompose into edge-disjoint directed cuts and records the dual cycle statement. Backman-Baker-Yuen Lemma 4.1.1 gives conformal disjoint signed-circuit decomposition for signed zero-one vectors in a regular matroid, and Ding Lemma 2.5 states the graph cycle and cocycle versions directly. These match the focal symmetric-difference use after restricting to cuts that avoid the added constraint arcs.

Verifiable source: [Ding, exact arXiv:2109.01930v4](#)

Corollary 3.7: The attributed generating-function identity and standard constructions match

PDF pages 5-6 · Corollary 3.7 and proof · exact arXiv:1909.01550v3 and official Analytic Combinatorics copy · arXiv:2605.16028v1

Applicable and sufficient

Archer-Gessel-Graves-Liang Equation (16) is the logarithmic form equivalent to the focal exponential identity. Flajolet-Sedgewick's labelled sequence and set constructions give respectively the reciprocal and exponential generating functions used in the proof. The focal application preserves the empty and one-vertex cases.

Verifiable source: [Archer, Gessel, Graves, and Liang, exact arXiv:1909.01550v3](#)

Gioan historical input: The author-hosted published article supports the attributed special cases

PDF pages 1, 3, and 10 · root-connectivity identity and reversal-class history · Gioan published pages 1355-1364 · arXiv:2605.16028v1

Applicable and sufficient

Gioan's exact author-hosted European Journal of Combinatorics article proves the relevant Tutte-polynomial counts: Theorem 3.4 and Proposition 4.7 with Corollary 4.8 give the rooted reachability and cocycle-class specialization, while Corollaries 4.11 and 4.13 give the cycle and cycle-cocycle class counts. Its Section 4 explicitly assumes a connected graph, which agrees with the connectivity qualification independently required for the focal introduction's spanning-tree wording.

Verifiable source: [Gioan, author-hosted European Journal of Combinatorics article](#)

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. [Official arXiv:2605.16028v1](#)
2. [Ding, exact arXiv:2109.01930v4](#)
3. [Archer, Gessel, Graves, and Liang, exact arXiv:1909.01550v3](#)
4. [Gioan, author-hosted European Journal of Combinatorics article](#)

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. *Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.*

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use $\$ \dots \$$ for inline mathematics and $\$ \$ \dots \$ \$$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.
- Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.
- Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.