MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

McShane-Rivin norm balls and simple-length multiplicities

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Paper	arXiv:2605.14574v2
Source version	Submitted 19 July, 2026; v1 submitted 14 May, 2026; announced May 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	May 14, 2026 (Theorem 1.1 and the v1 local consequences); July 19, 2026 (the new exact v2 flatness formula, strata, and rigidity)
Paper license	CC BY-NC-ND 4.0
Scope	Statements, proofs, and novelty only

SUMMARY

01	**Statements**	Correct
02	**Proofs**	Correct
03	**Novelty**	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The logarithmic-square lattice and simple-length multiplicity bounds, the rational-corner estimate, the exact irrational flatness formula and its level-set consequences, and the two stated corollaries are correct. The global estimate follows from the proved exponential normal-turn tail and a localized Jarník argument; the local formula follows from matching upper and lower turn estimates at irrational directions.

Theorem 1.1: Logarithmic-square boundary and simple-length counts

Page 4 · Theorem 1.1; proof on pages 17–18 · arXiv:2605.14574v2

Correct

For height cutoff H, the boundary points in rational directions of height at most H contribute $O(H^2)$ points. The complementary arcs have total normal turn $O_X(e^{-\kappa_X H})$ and total dilated arclength $O_X(L)$. Lemma 4.1 and Hölder's inequality therefore give

$$\#(L\partial B_X \cap \mathbb{Z}^2) \leq C_X L^{2/3} e^{-\kappa_X H/3} + O(H^2).$$

Choosing $H = \lceil 2\kappa_X^{-1} \log L \rceil$ proves the claimed $C_X(\log L)^2$ bound. Restricting to primitive vectors and identifying w with $-w$ gives the simple-length multiplicity estimate.

Verifiable source: Full paper, version 2

Theorem 1.2: Rational corners and exact irrational flatness

Pages 4–5 · Theorem 1.2; proofs on pages 19–29 · arXiv:2605.14574v2

Correct

For a primitive class w, the Fricke fan calculation gives a determinant of the endpoint supporting functionals equal in absolute value to

$$4\|w\|_X \log\coth\left(\frac{\|w\|_X}{2}\right),$$

which is comparable to $\mathrm{ht}(w)e^{-\|w\|_X}$; the matching upper bound follows by placing w in its height-$(\mathrm{ht}(w)-1)$ Farey gap. At an irrational direction the exponential tail makes the local turn tend to zero, so the supporting line is unique. The local turn has lower logarithmic decay rate $\tau_X(\beta)/\Omega(\beta)$, with the stated conventions at 0 and ∞. The graph-height estimate supplies the lower flatness bound, while rational corner atoms along a sequence realizing $\Omega(\beta)$ supply the matching obstruction. This yields exactly

$$\mathfrak{f}_X(\beta) = 1 + \frac{\tau_X(\beta)}{\Omega(\beta)}$$

in the finite positive case and the printed endpoint values. Continued-fraction constructions and the covering and Baire arguments then give all density, measure, category, and Hausdorff-dimension clauses.

Verifiable source: Full paper, version 2

Corollary 1.4: Finite flatness levels determine the marked torus

Page 6 · Corollary 1.4; proof on pages 29–30 · arXiv:2605.14574v2

Correct

On a dense overlap $\mathcal{F}_A(X) \cap \mathcal{F}_B(Y)$, the exact formula gives $\tau_X = c\tau_Y$ with $c = (A-1)/(B-1)$. Continuity extends this relation to all projective directions, hence $\|\cdot\|_X = c\|\cdot\|_Y$. McShane's identity rules out $c \neq 1$ by strict termwise comparison of the two convergent positive series. Equality of the resulting positive Fricke trace triple for a marked basis and its sum determines the marked cusped torus, giving $A = B$ and $X = Y$.

Verifiable source: McShane, simple-geodesic identity

Corollary 1.5: Markoff-fiber bound

Page 6 · Corollary 1.5 · arXiv:2605.14574v2

Correct

On the modular torus, a Markoff number m corresponds to length $L_m = 2\,\mathrm{arcosh}(3m/2)$. Thus $\log L_m \asymp \log\log(3m)$ outside a bounded initial range, and Theorem 1.1 gives

$$\#\lambda_M^{-1}(m) \leq C(\log\log(3m))^2.$$

Enlarging the constant covers the bounded range, including the smallest Markoff number.

02 / PROOFS

Proofs

Correct

The central proof chains are correct and complete. The Fricke recurrences give the required endpoint and corner estimates; Bowditch's sink theorem selects the larger root on every sufficiently high Farey gap; the turn tail sums correctly; and the convex-lattice, continued-fraction, and rigidity arguments preserve all stated constants, regimes, and quantifiers.

Theorem 2.3 and Lemma 2.5: Fricke identities and Bowditch root selection

Pages 8–10 · Theorem 2.3 and Lemma 2.5 · arXiv:2605.14574v2

Correct and complete

The positive trace labels satisfy the Vieta relations

$$x_{u+v} + x_{u-v} = x_u x_v, \qquad x_{u+v}x_{u-v} = x_u^2 + x_v^2.$$

Since every label is $2\cosh(\|w\|_X/2) > 2$, Bowditch's positive-Fuchsian sink conclusion applies. A height-H Farey gap with H beyond the three sink-label heights lies on the component opposite the sink; hence the arrow on its dual edge points toward $u - v$, which is exactly the inequality $x_{u+v} \geq x_{u-v}$ used later. The Farey-gap lemma correctly supplies determinant-one endpoint representatives and $\mathrm{ht}(u + v) > H$.

Verifiable source: Bowditch, Markoff triples and quasifuchsian groups

Propositions 3.1–3.2 and Theorem 3.3: Endpoint estimates and exponential normal-turn tail

Pages 13–16 · Section 3 · arXiv:2605.14574v2

Correct and complete

Solving the recurrence along $nu+v$ gives $x_{nu+v} = Ae^{na}+O_{u,v}(e^{-na})$, where $2a = \|u\|_X$, and therefore $\lambda_{u|v}(v) = 2\log A$. The displayed Vieta calculation proves $|2\log A - \|v\|_X| \lesssim_X e^{-\|u\|_X-\|v\|_X}$, and symmetrically at the other endpoint. Expressing the functional difference in the determinant-one dual basis incurs only the factor H. The two antipodal gap arcs have equal turn and each has turn at most π, so once the endpoint-normal angle is small the relevant swept interval is the smaller one. Summing over $O(H^2)$ gaps and absorbing the polynomial factor into a weaker exponential gives Theorem 3.3 with no omitted case.

Verifiable source: Full paper, version 2

Lemma 4.1 and proof of Theorem 1.1: Localized Jarník count

Pages 16–18 · Section 4 · arXiv:2605.14574v2

Correct and complete

Distinct consecutive lattice chords on a strictly convex arc have distinct primitive directions. In an angular sector of width θ, determinant integrality gives at most $O(\theta r^2 + 1)$ primitive directions of length at most r. Taking $r \asymp (N/\theta)^{1/2}$ forces at least half of N chord directions to be long, and the arclength budget yields $N \lesssim R^{2/3}\theta^{1/3} + 1$. Splitting a general arc into at most four normal sectors and applying Hölder preserves this estimate. The subsequent sum over the height-gap components uses the total arclength and total turn with the correct exponents.

Verifiable source: Full paper, version 2

Propositions 5.1–5.5 and Theorem 5.7: Corner atoms and the exact flatness exponent

Pages 19–27 · Sections 5.1–5.2 · arXiv:2605.14574v2

Correct and complete

The two fan coefficients satisfy $A_+A_- = \coth^2(\|w\|_X/2)$, so the supporting-functional determinant gives the lower corner bound; the containing Farey gap gives the upper bound. For local turn, $H_\beta(\rho)$ is bounded below by $Q^{-1}\log(1/\rho)$ whenever $Q > \Omega(\beta)$, while a sequence of rational directions attaining the limsup contributes corner atoms of size at least $\exp(-(\tau_X(\beta)+o(1))\,\mathrm{ht}(w))$. These bounds prove the three cases of Proposition 5.5. In graph coordinates, projective distance is comparable to $|x|$, graph height is controlled by local turn, and a corner of angle α at x_0 forces height at least $\alpha|x_0|/2$ at $3x_0/2$. Applying these estimates in both directions yields the exact supremal flatness order, including $\Omega = 0$ and $\Omega = \infty$.

Verifiable source: Full paper, version 2

Lemmas 5.8–5.10 and Corollary 5.11: Continued fractions, strata, and rigidity

Pages 27–30 · Sections 5.3–5.4 · arXiv:2605.14574v2

Correct and complete

For $\beta = [(1,b)]$, the usual best-approximation inequalities give

$$\Omega(\beta) = \frac{1}{\max\{1,|b|\}}\limsup_j \frac{\log q_{j+1}}{q_j}.$$

Prescribing the next partial quotient realizes every desired value of the resulting flatness order inside any projective interval. The exact-height count $O(h)$ makes every exponentially approximable set have Hausdorff dimension zero, while the open dense limsup construction gives $\{\Omega = \infty\}$ as a dense G_δ. These facts imply all metric and category statements. The final use of McShane's identity and the positive Fricke parametrization correctly removes the only possible scaling ambiguity between two marked norms.

Verifiable source: Full paper, version 2

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 2
2. McShane, simple-geodesic identity
3. Bowditch, Markoff triples and quasifuchsian groups

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.