MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Handling some Diophantine equation via Euclidean algorithm and its application to purely exponential equations

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Paper	arXiv:2604.18991v2
Source version	Submitted 21 July, 2026; v1 submitted 21 April, 2026; announced April 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	April 21, 2026
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains unverified proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The effective-finiteness theorem and the Euclidean-algorithm congruence are supported. No counterexample was found to the two classification theorems, but their decisive computer calculations are not supplied in a reproducible form, so those conclusions are not able to be verified from the paper and cited sources.

Theorem 1: Effective finiteness for fixed primes $c = 3 \cdot 2^r + 1$

Pages 3 and 7–11 · Theorem 1 and Section 5 · arXiv:2604.18991v2

Correct

The cited reductions leave only $E = 3$ and, outside an effectively bounded set, the system $a + b = c^z$, $a + b^Y = c^Z$ with bounded $Y \equiv 4 \pmod 6$. Lemmas 5.9–5.12 give $b \gg_c c^z$, whereas Lemma 5.8 and $b^Y < c^Z$ give $b < c^{(Y-1)z/Y}$. Hence $c^{z/Y} \ll_c 1$; bounded Y makes z effectively bounded, and then a, b are effectively bounded. This proves exactly the claimed effective finiteness.

Theorem 2: Classification for $c \in \{7, 13, 97\}$

Pages 4 and 13–35 · Theorem 2 and Sections 7–9 · arXiv:2604.18991v2

Not able to verify

No counterexample was found. The $c = 13$ case is independently covered by the earlier result that the paper explicitly cites. For $c = 7$ and $c = 97$, however, the exclusions ultimately depend on large finite searches whose complete inputs, executable code, and output certificates are not provided. The displayed pseudocode in Section 8.4 is an outline rather than a reproducible implementation, and decisive computed claims such as Lemma 9.7 and the final brute-force checks are stated without certificates. The available material is therefore insufficient to verify the full three-value classification.

Theorem 3: Pillai-equation classification for the listed 24 primes

Pages 4 and 35–36 · Theorem 3, Tables 1–2, and Section 10 · arXiv:2604.18991v2

Not able to verify

No counterexample was found. The paper reports that variants of earlier algorithms were run for every listed value of r, including about 52 hours of computation for the second stage, but supplies only summary tables and no complete algorithm, code, case log, or independently checkable certificate. Since these searches eliminate all remaining cases and are essential to the universal conclusion, the theorem cannot be verified from the supplied record.

Proposition 11.1: Congruence obtained from the polynomial Euclidean algorithm

Pages 37–41 · Proposition 11.1 and Lemmas 11.1–11.3 · arXiv:2604.18991v2

Correct

Factoring $X^m - X^n$ with $m - n = EN$ and applying the lifting-the-exponent identity gives Equations (11.3)–(11.4), including $y_2 > e$. The Bézout identity (11.2), evaluated at X and combined with those valuations, yields the claimed congruence modulo q^κ. The two alternatives defining κ agree with the lower bounds for y_1 proved in Lemma 11.3.

02 / PROOFS

Proofs

Contains unverified proofs

The proof of effective finiteness and the Euclidean-algorithm argument are complete. The proofs of the two explicit classification theorems omit the reproducible computational evidence needed to check their exhaustive searches; no mathematical contradiction in those conclusions was established.

Theorem 1: Reduction and effective-boundedness argument

Pages 7–11 · Lemmas 5.1–5.12 and proof of Theorem 1 · arXiv:2604.18991v2

Correct and complete

The congruence and valuation steps preserve the required coprimality hypotheses, and the Euclidean identity in Lemma 5.11 gives a genuine integer divisibility. Together with the cited effective bounds, the final comparison bounds z, then a and b, by effectively computable constants depending only on fixed c. No case used in the reduction is lost.

Theorem 2, case $c = 7$: The exhaustive sieves are not reproducible from the displayed outline

Pages 16–29 · Sections 7.2 and 8.4 · arXiv:2604.18991v2

Incomplete as written

The argument requires exhaustive searches over large, dynamically computed bounds. Section 7.2 describes a multistage procedure and Section 8.4 prints high-level pseudocode, but operations such as the valuation sieves and the final power test are not specified as executable code and no output certificate is included. Repeating the claimed searches, rather than filling a routine technical detail, is necessary to exclude the remaining solutions. A repair would publish the exact Magma source, version, inputs, and auditable outputs or provide independently checkable certificates.

Theorem 2, cases $c = 13, 97$: Decisive computed bounds and terminal checks lack certificates

Pages 29–35 · Section 9, especially Lemma 9.7 and proof of Theorem 2 · arXiv:2604.18991v2

Incomplete as written

Lemma 9.7 is obtained by checking every permitted Z on a computer, and the $c = 97$ proof then invokes further computer checks and a final brute-force computation. The manuscript does not provide code or output certificates for these steps. This is a verification limitation, not evidence that a computed bound or the theorem is false.

Theorem 3: The 24-parameter computation is summarized but not specified

Pages 35–36 · Section 10 and Tables 1–2 · arXiv:2604.18991v2

Incomplete as written

The proof says that almost the same algorithms as in earlier sections were performed for the listed values of r and reports aggregate running times. That description does not uniquely determine all computations, and the tables record intermediate bounds rather than certificates that every terminal case was rejected. Publishing a complete implementation and verifiable logs would address this audit finding.

Proposition 11.1: Valuation and Bézout-identity proof

Pages 37–41 · Equations (11.1)–(11.11) · arXiv:2604.18991v2

Correct and complete

The proof separates the two signs allowed by the generalized order, uses oddness of N to force $X^E \equiv 1 \pmod{q}$, and applies the standard odd-prime lifting-the-exponent formula to $I_{E,N}(X)$. Clearing denominators in the polynomial Bézout identity and reducing at the exact modulus supplied by Lemma 11.3 yields the stated result without an omitted case.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2604.18991v2

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.