MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, novelty, and source audit

Uniform Diophantine approximation with restrictions via total density of collections of subspaces

Leo Hong, Dmitry Kleinbock, Vasiliy Neckrasov

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Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Contains incorrect or incomplete source use

01 / STATEMENTS

Statements

Contains wrong statements

The total-density conclusions of Theorems 1.5, 1.6, 2.1, and 2.3 are correct after the verified proof repairs recorded below. Their advertised uniform-approximation consequence is false, however, when the exceptional family is allowed to consist of arbitrary proper analytic submanifolds: the invoked KMWW theorem requires closed nowhere dense exceptional sets. Restricting the claim to proper closed analytic submanifolds repairs every affected application.

Formula (1.6), Theorems 1.5–1.6, and Corollaries 1.8–1.9: The analytic exceptional sets must be closed

PDF pages 4–8 · formula (1.6), Theorems 1.5–1.6, and Corollaries 1.8–1.9 · arXiv:2603.25988v1

Incorrect as written · verified scope repair

The paper claims that the relevant uniform set remains uncountable and dense after deleting any countable family of proper analytic submanifolds of $M_{m,n}(\mathbb{R})$, while both its displayed Theorem B and the exact KMWW theorem require the forbidden sets to be closed nowhere dense. Enumerate open balls with rational centers and positive rational radii. Each ball is a proper real-analytic submanifold as a proper open subset, and the resulting countable family covers $M_{m,n}(\mathbb{R})$; the displayed set difference is therefore empty. Inserting 'closed' before 'analytic submanifolds' is the verified repair. Proper closed analytic submanifolds are nowhere dense and are respected by the affine resonant fibers, exactly as the paper checks immediately before formula (1.6). The total-density assertions themselves are unaffected.

Verifiable source: KMWW, Compositio Mathematica version of record

Theorems 2.1 and 2.3: The two geometric criteria yield property (TDS)

PDF pages 9–10 and 23–26 · Theorems 2.1 and 2.3 and their proofs · arXiv:2603.25988v1

Correct

For Theorem 2.1, the generic-dimension proposition makes the relevant product families dense, and the repaired local-intersection lemma promotes the prescribed closed pieces to a totally dense subcollection. For Theorem 2.3, the partition-sensitive finite convex-hull lemma reduces the direction sets to finitely many pieces, after which the same total-density proposition applies. Replacing the few unprimed direction sets by the primed sets just constructed and correcting the opposite-ball notation makes every hypothesis match. These repairs establish all three alternatives of Theorem 2.1 and the full convex-hull criterion of Theorem 2.3.

Verifiable source: Exact arXiv version 1

Theorem 1.6: Logarithmic density and two asymptotic halfspheres give a totally dense subcollection

PDF pages 5–6 and 11–12 · Theorem 1.6 and proof · arXiv:2603.25988v1

Correct

Logarithmic density supplies every required numerator-to-denominator ratio along each limiting direction. For radii tending to infinity, extract convergent pairs of admissible halfspheres. If two limiting supporting spheres differ, Theorem 2.1(a) applies. Otherwise select from every finite pair a halfsphere whose support differs from the common limiting sphere; points on its limit are approached by parameter directions off that whole sphere, and symmetrization gives Theorem 2.1(c). This verified compactness repair makes the dichotomy exhaustive and proves property (TDS). With 'closed' inserted in the exceptional-set clause, the two inhomogeneous corollaries follow from the stated augmented-matrix substitutions.

Verifiable source: Exact arXiv version 1

Theorem 1.5: The two product-restriction collections are totally dense

PDF pages 4–5 and 35 · Theorem 1.5 and proof · arXiv:2603.25988v1

Correct

In part (a), set $k = \min\{n-2, m\}$ and choose an $(m-k)$-dimensional subspace H_0 of the real span of P; the rank inequality guarantees its existence. The normalized parameter closure contains $H_0 \times \Phi$ for a k-subsphere Φ separated from the fixed direction, so Proposition 3.4 and Lemma 4.2 apply in their stated ranges. In part (b), choose the sign of an unbounded coordinate subsequence and separate that direction from both signs of the fixed direction. Bounded Hausdorff distance supplies all numerator ratios, Corollary 3.6 gives density, and Lemma 4.2 gives total density. Thus both total-density assertions are correct after the repairs.

Verifiable source: Exact arXiv version 1

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proofs are not complete as printed. The KMWW reduction is invoked outside its closedness and countability hypotheses; the proof of Theorem 1.6 assumes a non-exhaustive limiting-halfsphere dichotomy; Lemma 4.2 omits mixed coordinate cases; and the proof of Theorem 1.5 uses inadmissible dimensions and swapped normalizations. The first issue forces the statement restriction recorded above. The other defects have verified repairs, and the remaining extrema, primes, opposite balls, signs, and ranks have uniquely determined local corrections.

KMWW reduction behind formula (1.6): Two source hypotheses are not discharged in the printed reduction

PDF pages 3–8 · Theorem B and the 'hence' clauses following property (TDS) · arXiv:2603.25988v1

Incomplete as written · verified repair after restricting the statement

First, arbitrary analytic exceptional sets need not be closed or nowhere dense, so KMWW cannot imply the printed scope; this is repaired only by inserting 'closed' in every affected conclusion. Second, Theorem B requires a countable aligned collection, whereas the focal criteria allow arbitrary subsets P and Q. The latter omission has a verified repair. In the second-countable space $M_{m,n}(\mathbb{R})$, start with a countable subfamily whose union is dense. Recursively, for every selected fiber L and basis open set W meeting L, total density supplies a nonempty open set inside the closure of the eligible fibers meeting $L \cap W$; select countably many eligible fibers with union dense in that open set. The union over countably many stages is a countable totally dense subcollection. Finally, all relevant affine fibers have no isolated points, as checked in the Sources dimension, so the repaired uncountability step applies.

Verifiable source: KMWW, Compositio Mathematica version of record

Proof of Theorem 1.6: The two printed limiting-halfsphere cases are not exhaustive

PDF page 12 · proof of Theorem 1.6 · arXiv:2603.25988v1

Incomplete as written · verified repair

Compactness gives a limiting halfsphere, but the proof neither derives its two declared cases nor justifies replacing that halfsphere by its entire supporting sphere. Choose radii $C_j \to \infty$ and admissible pairs of halfspheres at those radii, then pass to limits of both supports and orientations. If two limiting supports differ, Theorem 2.1(a) applies. Otherwise all selected supports converge to one k-sphere S. From each finite pair choose a halfsphere whose support is not S and pass to a limiting halfsphere in S. Every point of that limit is approached by directions with norm exceeding C_j and lying outside S; symmetrization supplies the opposite halfsphere. Consequently

$$H \times S \subseteq \overline{E \setminus (H \times S)},$$

which is precisely Theorem 2.1(c). This proves the missing dichotomy without changing Theorem 1.6.

Verifiable source: Exact arXiv version 1

Lemma 4.2: The printed cases omit mixed coordinate separation

PDF pages 21–22 · proof of Lemma 4.2 · arXiv:2603.25988v1

Incorrect as written · verified repair

A parameter outside the two product balls need not have its direction coordinate separated from both signs or its numerator coordinate separated from both signs; one sign can be excluded by each coordinate. Fix $s \in \{1, -1\}$. For the max product metric used by the printed case split, if the direction is not already separated from $s\theta_0$, exclusion from the ball about sr_0 forces $\|p - sp_0\| > \varepsilon$; for any equivalent standard product metric the same argument works after replacing ε by a fixed smaller multiple. For $s = 1$ and $s = -1$, respectively, use

$$p - p_0 = Y(\theta - \theta_0) + (Y - Y_0)\theta_0$$

and

$$p + p_0 = Y(\theta + \theta_0) + (Y_0 - Y)\theta_0.$$

Since $\|Y - Y_0\| < \varepsilon/2$ and $\|Y\| < \rho_0$, both give $\|\theta - s\theta_0\| > \varepsilon/(2\rho_0)$ in the max-metric normalization. The angle lemma now applies sign by sign and proves the required local intersection.

Verifiable source: Exact arXiv version 1

Proof of Theorem 1.5: The normalized parameters and the sphere dimension are outside the printed ranges

PDF page 35 · proof of Theorem 1.5 · arXiv:2603.25988v1

Incorrect as written · verified repair

The proof fixes a pair in the wrong coordinate order, writes the normalization with the two coordinates reversed, and applies Proposition 3.4 with $k = n - 2$ and the full span of P, even when $n - 2 > m$ or that span has dimension larger than $m - k$. It also calls Φ an $(n-1)$-subsphere, which is the whole S^{n-1} and cannot satisfy the displayed positive-separation condition. Fix $(p_0, q_0) \in P \times Q$ and

$$r_0 = (p_0/\|q_0\|, q_0/\|q_0\|)\,.$$

In part (a), take $k = \min\{n - 2, m\}$, choose H_0 of dimension $m - k$ inside $\operatorname{span}_{\mathbb{R}} P$, choose a k-subsphere away from the fixed direction, and use $p/(j\|q\|)$ with a positive scaling integer j. The parameter closure then contains H_0 times that subsphere and Proposition 3.4 applies. In part (b), take the separation radius below half the distance from the limiting coordinate direction to each of $q_0/\|q_0\|$ and $-q_0/\|q_0\|$. These changes verify both branches.

Verifiable source: Exact arXiv version 1

Proposition 3.5 and Lemma 3.2: The separating functional needs a supremum and the constructed matrix has rank r

PDF pages 18–20 · Lemma 3.2 and converse proof of Proposition 3.5 · arXiv:2603.25988v1

Typo

Strict separation gives a functional with $\alpha^\top\theta < 0$ uniformly over the closed direction set. Thus the open set in Proposition 3.5 is defined by $\sup_\theta \alpha^\top\theta < 0$ and the positive constant is $C(A) = -\sup_\theta \alpha^\top\theta$; the two printed infima contradict the inequality used on the same page. In Lemma 3.2, adjoining $r - l$ new image vectors to the l fixed vectors makes the displayed augmented matrix have rank r, not rank $r - l$. Both corrections are mechanically fixed by the surrounding displays and alter no conclusion.

Verifiable source: Exact arXiv version 1

Proposition 4.3 and proof of Theorem 2.3: Opposite balls and the newly constructed primed direction sets are mistyped

PDF pages 22–26 · Proposition 4.3 and proof of Theorem 2.3 · arXiv:2603.25988v1

Typo

Every repeated pair $B_\varepsilon(r_0) \cup B_\varepsilon(r_0)$ in Proposition 4.3 has second member $B_\varepsilon(-r_0)$. In the proof of Theorem 2.3, after the recurrent set Θ_2' and the complementary finite set Θ_1' are defined, the proof must remove $H \times \Theta_2'$ and build N_0 from Θ_2', rather than reverting to Θ_2. With those uniquely determined replacements, the dense collections have exactly the hypotheses established in equations (5.4)–(5.8).

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

04 / SOURCES

Sources

Contains incorrect or incomplete source use

The exact KMWW theorem, Eggleston's Carathéodory theorem, and Boyd–Vandenberghe separation result are sufficient for their intended mathematical roles. The focal affine fibers immediately satisfy the repaired no-isolated-points condition needed for KMWW uncountability. The source use is nevertheless incomplete as printed because the paper applies KMWW beyond its closed-exceptional-set and countable-family hypotheses; countability has a verified skeleton repair, while closedness requires narrowing the stated conclusions.

KMWW Theorem 1.5 — uncountability hypothesis: Every focal affine fiber discharges the repaired no-isolated-points condition

PDF pages 3–8 and 35 · Theorem B and every focal uniform-approximation application · arXiv:2603.25988v1

Applicable after verified source correction

The exact Compositio theorem says that uncountability follows when each aligned set is either connected or has no isolated points, but the VOR audit gives a connected-singleton counterexample and verifies the corrected sufficient hypothesis 'has no isolated points'. The focal paper repeats the invalid connectedness alternative in Theorem B. Its actual uses do not enter the bad regime: in an ambient $M_{m,N}(\mathbb{R})$ every resonant set

$$L_{p,q} = \{A \in M_{m,N}(\mathbb{R}) : Aq = p\}$$

is an affine space of dimension $m(N-1)$. The hypotheses of Theorem 1.5 force $N \geq 2$, and the range $0 \leq k \leq N-2$ in Theorem 1.6 does the same. Corollary 1.8 applies that theorem to augmented matrices with $N = n+1 \geq 2$, while Corollary 1.9 assumes $n > 1$. Hence every aligned fiber in every proof-critical application has positive dimension and no isolated points. This immediate dimension check fully discharges the repaired KMWW condition.

Verifiable source: KMWW, Compositio Mathematica version of record

KMWW Theorem 1.5 — closedness and countability: The focal 'hence' clauses exceed two explicit source hypotheses

PDF pages 3–8 · formula (1.6), Theorems 1.5–1.6, and Corollaries 1.8–1.9 · arXiv:2603.25988v1

Incorrect or incomplete source use

KMWW requires a countable aligned collection and a countable family of closed forbidden sets respected by that collection. The focal paper allows arbitrary P and Q in several criteria and claims avoidance of arbitrary proper analytic submanifolds. A countable totally dense skeleton can be extracted by the recursive second-countability construction given in the Proofs dimension, so the first mismatch is repaired without changing a conclusion. The second mismatch cannot be suppressed: a countable cover by proper open analytic submanifolds makes the claimed complement empty. Restricting the exceptional family to proper closed analytic submanifolds exactly matches the source and repairs the applications.

Verifiable source: KMWW, Compositio Mathematica version of record

Eggleston, Chapter 2.2: Carathéodory's theorem is quoted with the correct dimension bound

PDF page 24 · Theorem C and proof of Theorem 5.1 · arXiv:2603.25988v1

Applicable and sufficient

The recalled result is the exact finite-dimensional Carathéodory theorem specialized to the origin: if $0 \in \operatorname{conv}(S) \subseteq \mathbb{R}^d$, then some subset of at most $d+1$ points already has the origin in its convex hull. The focal proof applies it once to obtain a base witness and at most once for each base witness lying in S_1. Their union has cardinality at most $(d+1)(d+2)$ and satisfies the stated deletion property. No stronger conclusion is attributed to the book, and the remainder of the partition-sensitive argument is proved internally.

Verifiable source: H. G. Eggleston, Convexity, Chapter 2.2

Boyd–Vandenberghe, Section 2.5.1: Strict separation applies to the compact convex hulls used in both branches

PDF pages 19 and 29–30 · Proposition 3.5 and the optimality proof · arXiv:2603.25988v1

Applicable and sufficient

The relevant direction set lies on the unit sphere, so its closure is compact and its convex hull is compact. If that convex hull omits the origin, strong separation from the singleton $\{0\}$ supplies a functional with a uniform strict sign. In Proposition 3.5 this yields $\sup_\theta \alpha^\top \theta < 0$ and an open matrix set uniformly separated from every associated affine fiber; in the optimality proof it yields the printed positive margin δ. Thus the cited theorem has exactly the hypotheses and strength required. The infimum in Proposition 3.5 is an internal notation typo, not a deficiency in the source.

Verifiable source: Boyd and Vandenberghe, Convex Optimization

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2603.25988v1
2. KMWW, Compositio Mathematica version of record
3. Exact arXiv version 1
4. H. G. Eggleston, Convexity, Chapter 2.2
5. Boyd and Vandenberghe, Convex Optimization

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- *Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.*
- *Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.*
- *Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.*

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.