MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, novelty, and source audit

Bijectivity of a generalized Pak-Stanley labeling

Olivier Bernardi, Neha Goregaokar

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Contains incorrect or incomplete source use

01 / STATEMENTS

Statements

Contains wrong statements

The right-inverse theorem and the sufficient bijectivity theorem for (m, ε)-arrangements are correct after the verified local repairs. Theorem 5.1 is false in its stated positive-n scope: for $n = 2$ and $S_{1,2} = \{2, 7\}$, the arrangement is transitive and its three regions are bijectively labeled, but it is not an (m, ε)-arrangement. The exhaustive $n = 3$ tests still support a possible corrected classification in a narrower scope, although the general converse sketches remain incomplete. Several foundational definitions and the $m = 0$ queue boundary also require explicit repairs.

Definitions 2.1–2.2: The parking-function domain and the off-diagonal arc set must be stated

PDF page 4 · Definitions 2.1–2.2 · arXiv:2603.24886v1

Unsupported as written · verified repair

Definition 2.2 calls $p = (p_1, \ldots, p_n)$ only a sequence. With no requirement that its entries be nonnegative integers, every sufficiently negative integer or real vector satisfies the displayed upper inequalities, while the algorithm of Section 3 can start with neither a zero nor an open-list entry and return an incomplete word. The intended and verified repair is $p \in \mathbb{Z}^n_{\geq 0}$. Definition 2.1 also quantifies over all $i, j \in [n]$, although $S^+_{i,i}$ is never defined; replacing this by $i \neq j$ gives the intended loop-free directed multigraph. Under these repairs, the sink arc makes each parking-function coordinate finite and the Section 3 algorithm has the required domain.

Verifiable source: Exact arXiv version 1

Definitions 1.3 and 2.7–2.8: Index and empty-set conventions are missing from the main scope

PDF pages 3 and 5–6 · Definition 1.3, Definition 2.7, Definition 2.8, and the setup of Section 3 · arXiv:2603.24886v1

Ill-defined as written · verified conventions

Definition 2.7 quantifies i, j, s, t but uses an unbound index k; Definitions 2.7, 4.3, and 4.5 also range into diagonal symbols $S^+_{i,i}$ that the notation never defines. Bernardi's imported definition uses distinct indices, and restricting the displayed conditions to the pairwise-distinct indices actually occurring in their S^+ terms is the verified repair. In addition, $m = \max \bigcup S^+_{i,j}$ is undefined for the empty arrangement, although every tuple of finite sets is allowed. Taking this maximum to be zero and declaring $[a; b] = \varnothing$ for $b < a$ makes Definition 1.3 represent the empty boundary with $m_k = 0$ and all relevant $\varepsilon_{i,k} = 1$. It does not by itself repair Section 3: when $m = 0$, Case 1 must append k to the open list only if $m > 0$, or Case 2 emits a nonexistent letter $\alpha^{(1)}_k$. A local-maximality clause with no qualifying adjacency must likewise be read vacuously.

Verifiable source: Bernardi, exact version of record

Theorem 3.2: The right-inverse identity survives the definition repairs

PDF pages 6–8 · Lemmas 3.1 and 3.5 and Theorem 3.2 · arXiv:2603.24886v1

Correct after verified local repairs

For a nonnegative directed-graph parking function, the zero/queue procedure cannot terminate with a positive coordinate: applying the parking inequality to the remaining positive index set gives the contradiction printed in Lemma 3.1. First-in-first-out processing preserves the sketch order, and the coordinate invariant in Lemma 3.5 counts exactly the source letters preceding each zero-letter. Evaluating that invariant when the zero-letter is emitted proves the right-inverse identity coordinate by coordinate. Two boundary repairs are necessary: the printed terminal value $-m$ must be $-(m+1)$, and Case 1 must enqueue k after $\alpha_k^{(0)}$ only when $m > 0$. Without the latter repair, the empty $n = 2, m = 0$ case with $p = (0, 0)$ emits a nonexistent level-one letter. With both repairs, exhaustive checks covered 742,656 parking-function inputs across all 33,280 three-index arrangements with $m = 1$ or $m = 2$, and a dedicated $m = 0$ regression check, with no failure.

Verifiable source: Exact arXiv version 1

Theorem 4.1: The sufficient bijectivity theorem uses only the sound half of Lemma 4.4

PDF pages 8–11 · Theorem 4.1, Lemmas 4.2 and 4.6, and the forward half of Lemma 4.4 · arXiv:2603.24886v1

Correct after verified local repairs

An (m, ε)-arrangement directly satisfies Properties X and Y. The proof of Theorem 4.1 then uses Y only in the sound direction $Y \implies M_S = L_S$, uses Lemma 4.6 to obtain $N_S = M_S$ from X, and invokes Bernardi's surjection through Lemma 4.2. It does not use the ellipsis-based converse $M_S = L_S \implies Y$. After the off-diagonal index and empty-interval conventions are inserted and the Section 3 algorithm queues a zero-level index only when $m > 0$, each required implication is verified and the explicit inverse is the repaired Section 3 right inverse.

Verifiable source: Exact arXiv version 1

Theorem 5.1: The transitive classification is false in the stated positive-n scope

PDF pages 11–13 · Theorem 5.1 and Lemma 5.3 · arXiv:2603.24886v1

Wrong as stated · explicit n=2 counterexample

Let $n = 2$ and $S_{1,2} = \{2, 7\}$, so $S^+_{1,2} = \{2, 7\}$ and $S^+_{2,1} = \varnothing$. Bernardi transitivity is vacuous because it quantifies pairwise-distinct triples. The arrangement consists of two parallel hyperplanes and has three regions. Its graph has two arcs $1 \to 2$ and one sink arc from each vertex, hence $\mathrm{Park}_S = \{(0, 0), (1, 0), (2, 0)\}$; direct evaluation of λ gives exactly these three distinct labels, so condition (a) holds. After the necessary distinct-index repair, Properties X and Y are also vacuous, so (c) holds. But an (m, ε)-arrangement would require $S^+_{1,2} = [1; m_2 - \varepsilon_{1,2}]$, an initial interval, which $\{2, 7\}$ is not. Thus (a) and (c) do not imply (d), disproving Theorem 5.1 in its stated scope. Exhaustive enumeration still found the claimed equivalences for all 512 arrangements with $n = 3, m = 1$ and all 32,768 with $n = 3, m = 2$, but restricting to $n \geq 3$ would only remove this counterexample; the incomplete converse sketches in Lemmas 4.4 and 5.3 would still need a general proof.

Verifiable source: Exact arXiv version 1

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The Section 3 algorithm has a repairable off-by-one assertion and an $m = 0$ queue error. The final reconstruction of the (m, ε) parameters has an unbound witness and boundary error and, more decisively, cannot prove the false $n = 2$ scope of Theorem 5.1. The converse sketch constructions in Lemmas 4.4 and 5.3 also omit the general extension and adjacency checks on which their contradictions depend; one displayed membership claim in Lemma 5.3 is false at shift zero. Finite exhaustive checks support the repaired claims at $n = 3$, but do not repair the theorem's stated scope or replace the missing arbitrary-size proof.

Lemma 3.1: The terminal queue coordinate is off by one

PDF pages 6–7 · first paragraph of the proof of Lemma 3.1 · arXiv:2603.24886v1

Incorrect as written · verified repair

An index is decremented from zero to -1 when its zero-letter is emitted, and once more after each of its m positive-level letters. Its terminal value is therefore $-(m+1)$, not $-m$ as printed; Figure 4 itself ends at -2 when $m = 1$. Replacing $-m$ by $-(m+1)$ makes the ensuing conclusion exact: every letter at levels $0, 1, \ldots, m$ appears once. There is a separate $m = 0$ control-flow defect: Case 1 unconditionally appends the index to the open list, so Case 2 later emits $\alpha_k^{(1)}$ even though a $(0, n)$-sketch has only zero-level letters. Appending only when $m > 0$ repairs this boundary. The contradiction for the remaining positive set and the local-maximality check are then valid.

Verifiable source: Exact arXiv version 1

Lemma 4.4: The two converse witnesses are specified only by ellipses

PDF page 9 · converse half of Lemma 4.4, Cases 1 and 2 · arXiv:2603.24886v1

Incomplete as written · unresolved in full generality

Failure of Property Y is answered by displaying fragments such as a descending zero-letter block followed later by two prescribed adjacencies. In Case 2, the stated block before $\alpha_k^{(0)}$ includes every $\alpha_p^{(0)}$ with $p > k$ and $p \neq j$; because the case assumes $i > k$, this already includes $\alpha_i^{(0)}$, which the displayed word then writes a second time. The local repair is to exclude both $p = i$ and $p = j$ from that first block. Even after this correction, the proof does not define the omitted letters, verify all three sketch axioms, or check every zero-letter adjacency required for membership in L_S. Those points are essential: an arbitrary word containing the fragment need not be a sketch. Exhaustive construction over every $(1, 3)$- and $(2, 3)$-sketch confirms the equivalence $Y \iff M_S = L_S$, and generic prefixes of the advertised kind had unique completions for $n = 3, 4$ and $m = 2$. This is strong support, but no general completion argument is supplied in version 1, so the converse remains unresolved as printed.

Verifiable source: Exact arXiv version 1

Lemma 4.6: The injectivity induction is valid after the distinct-index convention

PDF pages 9–10 · proof of Lemma 4.6 and equation (1) · arXiv:2603.24886v1

Correct after verified index repair

At the first differing position, two positive-level letters would force contradictory orders on their predecessors. Thus one word emits a zero-letter while the other delays it. Equality of the corresponding label coordinate excludes every contributing letter in the intervening segment, and Property X propagates the zero-level ordering backward through that segment to a contradiction. When the propagated index equals the fixed index, the needed alternative is immediate rather than an application of Property X. Restricting all S^+ symbols to unequal indices makes this implicit equality case and the displayed induction well-defined.

Verifiable source: Exact arXiv version 1

Lemma 5.3: The positive-shift witness contains a false membership claim and unproved completions

PDF pages 12–13 · second case in the proof of Lemma 5.3 · arXiv:2603.24886v1

Incorrect and incomplete · local repairs verified, general construction unresolved

From Property Y the proof claims $s-1 \in S^+_{\ell,k}$ for every $\ell \neq j$. This is false when $s = 1$ and $\ell < k$: zero is represented by the baseline clause in Triple_S, not by membership in $S^+_{\ell,k}$. The exact repair is the weaker statement $(\ell, k, s-1) \in \mathrm{Triple}_S$: for positive $s-1$ it follows from Y; at zero it follows either from the baseline order $\ell < k$ or, when $\ell > k$, from Y. Also, $n^* = \max([n] \setminus \{i, j, k\})$ is undefined for $n = 3$ and the corresponding block must be declared empty. These repairs justify the advertised boundary adjacencies. The proof still does not spell out the omitted suffixes, verify every local-maximality condition, or derive all equal label coordinates for arbitrary n, m. The complete $n = 3, m \leq 2$ enumeration found no failure, but the general witness verification remains unresolved.

Verifiable source: Exact arXiv version 1

Theorem 5.1, (c) implies (d): The parameter reconstruction has local errors and cannot cover n=2

PDF page 13 · final three paragraphs of the proof of Theorem 5.1 · arXiv:2603.24886v1

Incorrect as written · theorem false at n=2

After fixing k, the proof defines the least s through $S^+_{i,k}$ without choosing or quantifying i, and then uses that same free index throughout. A local repair is to choose a witness pair (i, s) with globally minimal admissible s, where $i \neq k$ and either $s > 0$ or $i > k$. In the branch where no set contains the boundary value, the printed choice $m_k = s - 1$ is negative when $s = 0$; the boundary repair is $m_k = s$ and $\varepsilon_{\ell,k} = 1$ for every $\ell \neq k$. The displayed monotonicity sentence is also backwards and should use the contrapositive of X, and $S_{\ell,k}$ is missing a plus sign. These corrections repair the printed local manipulations only where the argument has a third index. They cannot close the stated implication: for $n = 2$ and $S_{1,2} = \{2, 7\}$, X and Y are vacuous while the shift set is not an initial interval. The proof's attempted contradiction after finding a later shift silently needs an index $j \neq i, k$, which does not exist. A valid theorem must first narrow or otherwise amend its scope, and the remaining $n \geq 3$ reconstruction must still be checked together with Lemmas 4.4 and 5.3.

Verifiable source: Exact arXiv version 1

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

04 / SOURCES

Sources

Contains incorrect or incomplete source use

Bernardi's 2018 paper is the only direct proof-critical non-book source. Its Section 8 results have the strength the focal proofs need, but the focal pinpoint for the region/sketch bijection names the wrong proposition, and the exact journal proof contains a defective inverse construction repaired only in arXiv:1604.06554v4. The focal transitivity definition also drops Bernardi's distinct-index quantifier. After those repairs, the inherited branch is mathematically applicable; its deeper proof-critical inputs terminate at Zaslavsky's monograph and Stanley's book.

Bernardi region/sketch correspondence: The focal pinpoint is wrong and the cited journal construction needs the later repair

PDF page 5 · Lemma 2.6 and citation [2, Proposition 8.1] · arXiv:2603.24886v1

Incorrect or incomplete source use · verified repair

Bernardi Proposition 8.1 is not the region/sketch bijection; it maps annotated sketches to labeled $(m+1)$-ary trees. The region/sketch bijection is established by properties (i)–(iv) immediately before that proposition on journal pages 503–504. Moreover, the version-of-record recipe in property (iv) can create equal consecutive values: for $n=3, m=1$, the valid sketch $\alpha_1^{(0)}\alpha_2^{(0)}\alpha_1^{(1)}\alpha_3^{(0)}\alpha_2^{(1)}\alpha_3^{(1)}$ gives $z_4 = z_5 = 3/2$. Bernardi's arXiv version 4 repairs the recipe by using decreasing increments 2^{-p} for newly initialized coordinates while retaining the version of record's already-correct nonzero branch for later levels. That repair verifies the imported bijection, but the focal citation should point to the preceding properties and disclose the corrected source form.

Verifiable source: Bernardi, corrected arXiv version 4

Bernardi Theorem 8.8 branch: The locally maximal surjection and transitive bijection have the required scope

PDF pages 5–6 and 8–13 · Theorem 2.9 as used in Theorems 3.2, 4.1, and 5.1 · arXiv:2603.24886v1

Applicable and sufficient after the Section 8.1 repair

Bernardi Lemmas 8.12–8.13 identify each arrangement region with an equivalence class of Catalan subregions and translate local maximality into the corresponding tree condition. Their proof gives surjectivity for arbitrary S. When S is transitive, Bernardi Theorem 4.6 supplies equal finite cardinalities, so the map is bijective. These are exactly the two assertions quoted in focal Theorem 2.9. The focal paper's use is therefore sound once the earlier region/sketch construction is taken from the corrected arXiv version.

Verifiable source: Bernardi, Advances in Mathematics version of record

Transitivity definition: The focal transcription omits the source's distinct-index domain

PDF page 5 · Definition 2.7 · arXiv:2603.24886v1

Incomplete source transcription · verified repair

Bernardi Definition 4.3 quantifies over distinct indices a, b, c. Focal Definition 2.7 quantifies only i, j and then uses an unbound k; without distinctness it also calls undefined diagonal sets. Quantifying pairwise distinct i, j, k is both source-faithful and the domain used by the later proofs. The same explicit restriction should be propagated to Properties X and Y whenever their displayed S^+ terms would otherwise be diagonal.

Verifiable source: Bernardi, Advances in Mathematics version of record

Inherited counting branch: The transitive cardinality input terminates at two published monographs

PDF pages 5–6 and 11–13 · inherited use of Bernardi Theorem 8.8 in Theorem 2.9 and Theorem 5.1 · arXiv:2603.24886v1

Sources complete for the marked branch

Bernardi's bijectivity upgrade uses Theorem 4.6. Its proof-bearing chain passes through the signed region formula, where Zaslavsky's characteristic-polynomial evaluation converts the coboundary identity to a region count, and through Claim 7.6, where Stanley's Chapter 5.3 plane-tree/path encoding and cyclic-conjugation count evaluate the tree factor. Zaslavsky's Memoirs AMS monograph and Stanley's Enumerative Combinatorics, Volume 2 are published book terminals, so no further graph recursion is required. Contextual citations in the focal paper, including Mazin–Miller's alternative graphical proof and the earlier surjectivity result that Section 3 reproves, do not affect the marked claims.

Verifiable source: Bernardi, exact version of record

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2603.24886v1
2. Exact arXiv version 1
3. Bernardi, exact version of record
4. Bernardi, corrected arXiv version 4

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.
- Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.
- Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.