MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, novelty, and source audit

The geometry of a counting formula for deformations of the braid arrangement

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Source version	Submitted 25 March, 2026; announced March 2026.
Audit date	August 23, 2026
Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Sources correct and complete

01 / STATEMENTS

Statements

Contains wrong statements

The central contribution-one theorem and the final factorization of a tree contribution are correct after the verified proof repairs recorded below. One ancillary assertion is false: a minimal circular interval family containing a singleton need not have at least two connected components. The statement is unused and is repaired by replacing $t \geq 2$ with $t \geq 1$. The all-empty arrangement also needs the convention $m = 0$ because its displayed maximum is otherwise undefined.

Theorem 3.1: Every region has signed contribution one

PDF pages 6–7 and 15–16 · Theorem 3.1 and proof · arXiv:2603.24885v1

Correct after verified proof repairs

The marked-tree face bijection restricts to the S-boxed trees attached to a fixed region, and the number of boxes is the dimension of the corresponding relatively open face. Those faces form a finite cell decomposition of the region. Using compactly supported Euler characteristic gives

$$\sum_{F}(-1)^{n-\dim F} = (-1)^n \chi_c(R) = (-1)^n(-1)^n = 1,$$

because the open convex region R is homeomorphic to $\mathbb{R}^n$. The repaired Lemma 3.17 and this sign convention prove the theorem for every nonempty arrangement datum; adopting $m = 0$ covers the empty datum as well.

Verifiable source: Exact arXiv version 1

Proposition 4.19: The circular-component factorization of a tree contribution is correct

PDF pages 21–22 · Proposition 4.19 and proof · arXiv:2603.24885v1

Correct after verified local completions

After identifying incident faces with the proper subsets of the affine-alcove facet set, the forbidden interval conditions separate over the circular connected components J_i. If the complement S is nonempty, summing $(-1)^{|D^*|}$ over its subsets vanishes; if it is empty, the remaining independent sums multiply to $\prod_i w(J_i)$. This is exactly the displayed dichotomy. The proof remains valid when there is only one component.

Paragraph after Example 4.15: A singleton interval does not force two circular components

PDF page 20 · paragraph immediately before Definition 4.16 · arXiv:2603.24885v1

Incorrect · verified local repair

The manuscript infers $t \geq 2$ solely because the inclusion-minimal interval family $\mathcal{I}_S$ contains a singleton. This implication is false: $\mathcal{I}_S = \{\{1\}\}$ has one circular connected component. The configuration can also occur in the paper's geometric setting; for $n = 2$, take a deformation whose only hyperplane is $x_1 - x_2 = -2$ and the 2-Catalan alcove $1 < x_1 - x_2 < 2$. The deformation hyperplane is not face-supporting there, while exactly the outer facet is nonseparating, so the minimal interval family is a singleton. Replace $t \geq 2$ by $t \geq 1$. No later argument uses the stronger inequality.

Verifiable source: Exact arXiv version 1

Definitions 2.6 and 3.1: The empty arrangement needs a maximum convention

PDF pages 5–6 · Definition 2.6 and Theorem 3.1 · arXiv:2603.24885v1

Minor scope defect · verified repair

The theorem allows every collection of finite sets $S_{a,b}$, including the case in which all of them are empty, but then $m = \max\{|s| : s \in S_{a,b}\}$ is undefined. Set $m = 0$ when the union of all intercept sets is empty. The empty arrangement has one region, its tree sum reduces to the same Euler-characteristic identity, and all subsequent definitions are then meaningful.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The main proof is not valid exactly as printed. Lemma 3.17 confuses edge and path hyperplanes, reverses orientations and indices, and infers absence of a summed hyperplane only from absence of its individual edge hyperplanes. The two Euler-characteristic arguments use ordinary Euler characteristic with the compact-support sign. Section 4 also omits the onto parts of two affine-alcove identifications. All of these defects have direct verified repairs, and the repaired proof establishes the central theorem and final factorization.

Lemma 3.17: The restriction to S-boxed trees needs a path-sum argument

PDF pages 13–15 · proof of Lemma 3.17 · arXiv:2603.24885v1

Incorrect and incomplete as written · verified repair

Several printed steps cannot be used literally. The preliminary zero-child condition says $v < 0\text{-child}(u)$ while assuming $v = 0\text{-child}(u)$; it must say $u < v$. The inverse notation later writes $\alpha^{-1}(T, B)$ although the input is the marked tree (T, μ); it must be $\alpha^{-1}(T, \mu) = (T, B_\mu)$ (and the earlier β_μ label is likewise B_μ). The first implication reverses the path endpoints and its $S^-_{i,j}$ orientation. The converse writes $x_j - x_i$ where the vertices are v_j, v_i, ranges j up to n instead of the path length, and equates an intersection of edge hyperplanes with one hyperplane. Most importantly, from the fact that each marked edge hyperplane is absent from A_S it concludes that the summed hyperplane is absent, which does not follow. The repair is direct. For path positions $p < q$, put

$$t = \sum_{r=p}^{q-1} \mathrm{lsib}(v_{r+1}) \geq 0.$$

The marked equalities imply containment of the face in $\{x_{v_q} - x_{v_p} = t\}$. When $t = 0$, the marked-zero rule gives $v_p < \cdots < v_q$; with that orientation, the definition of $S^-_{v_p,v_q}$ says precisely that $t \in S^-_{v_p,v_q}$ iff this summed hyperplane belongs to A_S. Because $F \in \mathcal{F}_S$ excludes containment in every hyperplane of A_S, the sum is not in S^-. Conversely, if $F \notin \mathcal{F}_S$, orient the containing arrangement hyperplane so its displacement is nonnegative. The inverse marked tree supplies the corresponding marked cadet subpath, whose sum belongs to S^-, so its box is not an S-cadet sequence. This proves exactly $\beta_m(U_S) = \mathcal{F}_S$; the remaining incidence argument restricts the bijection to each region and the component count gives the dimension.

Verifiable source: Exact arXiv version 1

Corollary 3.10 and proof of Theorem 3.1: The finite open-cell sum is a compactly supported Euler characteristic

PDF pages 10–11 and 15–16 · proofs of Corollary 3.10 and Theorem 3.1 · arXiv:2603.24885v1

Incorrect convention as written · verified repair

The region is generally unbounded and is partitioned into relatively open, noncompact cells. Ordinary Euler characteristic is not finitely additive over that partition with the displayed signs. Use compactly supported Euler characteristic. A relatively open d-cell has $\chi_c = (-1)^d$, and an open convex n-region has $\chi_c = (-1)^n$. Therefore

$$\sum_F (-1)^{n-\dim F} = (-1)^n \sum_F (-1)^{\dim F} = (-1)^n \chi_c(R) = 1.$$

This fixes both occurrences and preserves every conclusion.

Lemmas 4.6–4.7, Remark 4.8, and Proposition 4.12: The affine-alcove face model needs its missing onto arguments

PDF pages 18–20 · Lemmas 4.6–4.7, Remark 4.8, and Proposition 4.12 · arXiv:2603.24885v1

Incomplete as written · verified repair

Lemma 4.6 fixes one supporting hyperplane and then states an equivalence that only holds after quantifying over every hyperplane containing F. Lemma 4.7 checks that the affine map sends a relatively bounded m-Catalan chamber into the standard alcove but does not prove equality. Relative boundedness rules out every outer interval, so each pairwise difference lies between consecutive integer walls already present in the m-Catalan arrangement; the chamber is therefore one affine type-A alcove, and the permutation/integer translation maps it onto the standard alcove. Remark 4.8's independence of m follows by appending only rightmost leaf children: cadet edges, left-sibling counts, and S-boxings do not change. Finally, every proper subset of the n standard-alcove facet equations has a unique nonempty relatively open face, while the full set is inconsistent. This supplies the omitted surjectivity of τ in Proposition 4.12 and completes the Boolean-face identification used by Proposition 4.19.

Local notation in Lemma 3.17: The final region restriction uses two stale symbols

PDF page 15 · final paragraphs of Lemma 3.17 · arXiv:2603.24885v1

Typo

The general-deformation proof refers to a region of A_G, ends with $\mathcal{F}_R$, and restricts an undefined bare β. These are copied or stale symbols. They must be a region of A_S, the set $\mathcal{F}_S(R)$, and the already defined map β_S, respectively, so the final line is $\beta_R = \beta_S|_{U_S(R)} : U_S(R) \to \mathcal{F}_S(R)$. The surrounding definitions determine every replacement uniquely.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

04 / SOURCES

Sources

Sources correct and complete

Every proof-critical external result was traced in its exact cited form. Bernardi's 2025 Theorem 3.8 supplies precisely the marked-tree face bijection used in Theorem 3.12; Bernardi's 2018 region bijection is also used directly and recursively. The 2018 version of record has a real inverse-construction defect, but its corrected arXiv version 4 repairs the exact theorem. Levear's Theorem 14 is an independently sufficient second source for the m-Catalan face bijection, not an additional logical prerequisite. No missing proof-critical citation was found.

Theorem 3.12 via Bernardi 2025: The marked-tree face theorem matches exactly

Focal PDF pages 12–15 · Theorem 3.12 and Lemma 3.17 · arXiv:2603.24885v1; Bernardi 2025 PDF pages 6 and 21–28 · Theorem 3.8

Correctly invoked

The m-Catalan arrangement is strongly transitive, so Bernardi's Theorem 3.8 applies. Its marked edges encode equality along cadet edges, its unmarked inequalities are the same Ψ formula reproduced by the focal paper, and its number of marked edges is codimension. Since a boxing with $|B|$ cadet components has $n - |B|$ marked edges, the face dimension is exactly $|B|$, which is the statistic required by Lemma 3.17.

Verifiable source: Olivier Bernardi, arXiv:2503.00662v1

Direct and recursive Bernardi 2018 input: The exact region theorem is valid after its documented repair

Focal PDF page 6 and Lemma 3.17 on page 15 · Theorem 2.9 · arXiv:2603.24885v1; Bernardi 2025 PDF pages 3 and 21–22 · Theorem 2.9 and proof of Theorem 3.8; Bernardi 2018 journal pages 503–513

Correct after verified source repair

The focal paper uses Bernardi's tree-to-Catalan-region map directly to define T_R and to identify the incident unmarked region of a face. Bernardi 2025 also invokes the same theorem on every restricted flat. In the exact 2018 version of record, property (iv)'s fixed increment can create equal consecutive z-values. ArXiv:1604.06554v4 replaces it by 2^{-p} and changes the second branch to $s \neq 0$, a repair verified against the downstream Proposition 8.1 and Theorem 8.8 proof. Theorem 8.8's surjectivity is internal; its transitive injectivity uses Theorem 4.6, whose proof-critical published stopping sources are Zaslavsky's Memoirs monograph and Stanley's Enumerative Combinatorics, Volume 2.

Verifiable source: Bernardi, corrected arXiv:1604.06554v4

Levear 2021 alternative: The independently cited Catalan-face bijection also supports Theorem 3.12

Focal PDF page 12 · Theorem 3.12 citation [4, Theorem 14] · arXiv:2603.24885v1; Levear version of record, Theorem 14 and Sections 2.2–3

Correct alternative source

Levear's exact open version of record proves a dimension-graded bijection between faces of the extended m-Catalan arrangement and decorated $(m+1)$-ary trees. After the notation translation recorded by Bernardi 2025, it gives the same theorem used by the focal paper. Because Bernardi 2025 already supplies the full required statement and the focal proof does not combine separate hypotheses from the two papers, Levear is recorded as an independently sufficient alternative rather than a second mandatory graph edge.

Verifiable source: Duncan Levear, Electronic Journal of Combinatorics 28(4) (2021), P4.29

Remaining bibliography: Contextual and reproduced results are correctly excluded from the proof graph

Focal PDF pages 1–2, 5–8, 16, and 22–23 · introduction, background, Section 4 motivation, and references · arXiv:2603.24885v1

Complete for the marked proof scope

The Bernardi signed-count theorem is recovered as a consequence rather than assumed in proving Theorem 3.1. Bisain–Hanson supplies context and an earlier tree-contribution algorithm but no step of the new proof. Orlik–Terao, Postnikov–Stanley, Stanley's survey, and Zaslavsky's theorem are background citations in the focal manuscript; no unproved step in Lemma 3.17 or Proposition 4.19 requires them directly. Their omission as direct focal edges therefore reflects logical dependence, not missing bibliography.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2603.24885v1
2. Exact arXiv version 1
3. Olivier Bernardi, arXiv:2503.00662v1
4. Bernardi, corrected arXiv:1604.06554v4
5. Duncan Levear, Electronic Journal of Combinatorics 28(4) (2021), P4.29

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.

- Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.

- Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.