

Correctness, proof completeness, and novelty audit

Fractional parts of powers of negative rationals

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

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01 / STATEMENTS

Statements

Correct

The lower bound for the oscillation of the weighted generating sums, its sharpness construction, and the resulting lower bound for fractional parts are correct. A false auxiliary lemma in the printed proof has a verified repair that applies at every use, so it does not change the validity of the main theorems.

Theorem 1.2: Sharp oscillation bound for weighted generating sums

Pages 2–7 · Theorem 1.2 and its proof · arXiv:2603.16794v3

Correct

Assuming that all limit values lie in an interval (A, B) of length less than $1 + r - r^2$, equation (2.3) first forces the sequence to use two consecutive integer values. Its applications to short blocks then exclude 010 and 101 and force the intervening zero-block lengths to belong to $\{k, k + 2\}$ for one even k . Because the resulting block-length word is not ultimately periodic, the corrected Lemma 2.1 supplies arbitrarily long common continuations with different preceding lengths. Equation (2.3) then gives $(1 + r^m)(B - A) > 1 + r - r^2$ with $m \rightarrow \infty$, a contradiction. The Sturmian balanced-word construction proves the matching upper bound for all shifted sums, so equality is attained for every $0 < r < 1$.

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Theorem 1.1: Fractional-part lower bound

Pages 1–2 and 8–9 · Theorem 1.1 and Section 3 · arXiv:2603.16794v3

Correct

Set $x_n = \lfloor \xi(-p/q)^n + \eta \rfloor$, $y_n = \{\xi(-p/q)^n + \eta\} - \eta$, and $s_n = -px_n - qx_{n+1}$. The identity $x_n + y_n = \xi(-p/q)^n$ gives $s_n = py_n + qy_{n+1}$, so (s_n) is bounded and the shifted generating series telescopes to $S_n(-q/p) = py_n$. Under the theorem's hypothesis that $\xi \notin \mathbb{Q}$ or $p/q \notin \mathbb{Z}$, the cited lemma rules out ultimate periodicity of (s_n) . Applying Theorem 1.2 with $r = q/p$ and dividing its oscillation bound by p gives exactly the displayed conclusion.

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02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The central arguments are correct after one necessary, verified repair: Lemma 2.1 is false for arbitrary infinite words and must assume that the word is not ultimately periodic. Every invocation already concerns a word proved to have that property. Two further notation errors are unambiguous typos and do not affect any conclusion.

Lemma 2.1: The left-special-factor lemma is missing a necessary hypothesis

Page 3 · Lemma 2.1 · arXiv:2603.16794v3

Incorrect as written · verified repair

As stated, the assertion that every infinite word over a finite alphabet has a left-special factor of every length is false: a constant infinite word has none. The proof invokes the strict Morse–Hedlund inequality, which requires that the word not be ultimately periodic. Adding the hypothesis that W is not ultimately periodic makes the argument valid. This is not merely a typographical issue because the printed lemma is a false mathematical statement. The repair is sufficient for the paper: each use is with the sequence S or Z , and the surrounding argument has already proved that the relevant word is not ultimately periodic.

Verifiable source: [Full paper, version 3](#)

Theorem 1.2 and Theorem 1.1: The main proofs are complete after the Lemma 2.1 repair

Pages 4–9 · Sections 2–3 · arXiv:2603.16794v3

Correct and complete after verified repair

The positive- and negative-parameter oscillation estimates correctly control the geometric tails, the equality construction gives both extremal values, and the final telescoping identity converts the negative-parameter estimate into the fractional-part theorem. No missing case remains once Lemma 2.1 is read with its necessary non-ultimate-periodicity hypothesis. In particular, the repair does not require changing either main statement.

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Proposition 2.3: Wrong equation number in the sharpness sentence

Page 4 · Proposition 2.3 · arXiv:2603.16794v3

Typo

The phrase saying that equality holds in (1.3) should refer to (2.4), the positive-parameter inequality stated immediately above. The subsequent Sturmian-word proof establishes the (2.4) equality, so the intended reference is unambiguous and no mathematical claim changes.

Sharpness construction: Index and symbol are interchanged in one sentence

Page 7 · End of the proof of Theorem 1.2 · arXiv:2603.16794v3

Typo

The sentence “ $s_j = 1$ if and only if $s_j = \sigma_n$ or $s_j = \sigma_n - 1$ ” mismatches a symbol value with index values. It should read “ $s_j = 1$ if and only if $j = \sigma_n$ or $j = \sigma_n - 1$.” The construction and all following sums use this intended indexing, so the correction is purely typographical.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: [arXiv:2603.16794v3](#)
2. Full paper, version 3

arXiv acknowledgement

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be *Correct* in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an *Incorrect* theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.