MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On some results of Korobov and Larcher and Zaremba's conjecture

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Paper	arXiv:2603.14116v2
Source version	Submitted 24 June, 2026; v1 submitted 14 March, 2026; announced March 2026.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	March 14, 2026
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

No counterexample was found to the central conclusions. Theorems 6–7 are not able to be verified because their proof chain contains unresolved gaps even in the prime case, while their composite branches also use false counting inputs. The composite branch of Theorem 8 is likewise not verified. Corollary 1 is correct by a later complete proof.

Theorems 6–7: The full stated conclusions are not established by the printed proof

Pages 4–5 and 25–36 · Theorems 6–7, Lemma 32, Proposition 35, and Sections 5.2–5.3 · arXiv:2603.14116v2

Not able to verify

The prime and composite arguments use the construction supplied by Lemma 32 and Proposition 35. Equation (107) in Lemma 32 drops an uncontrolled term, and Proposition 35 chooses coefficient bounds only after seeing the output of a lemma that requires those bounds as fixed inputs. No uniform or simultaneous selection argument repairs these obligations. The composite branches additionally depend on Lemmas 14–15 and Corollary 16, whose full-ring main terms are formally false. These defects prevent verification but do not provide a counterexample to either theorem.

Theorem 8: The stated composite-modulus scope is not established

Pages 6 and 36–37 · Theorem 8 and Section 5.4 · arXiv:2603.14116v2

Not able to verify

The proof applies Corollary 16 and Lemma 20 to composite moduli. Their derivation uses a main term that omits the density of units, and the corrected factor is not propagated through the displayed lower bound and parameter choices. This is confined to the affected composite scope: it is not a counterexample to Theorem 8 and does not by itself classify the prime branch as incorrect.

Corollary 1: The Zaremba existence statement is independently verified

Page 2 · Corollary 1 and Equation (5) · arXiv:2603.14116v2

Correct

Xin Zhang's later Theorem 1.2 proves the stronger assertion for every denominator using a corrected composite-modulus input. Corollary 1 therefore passes the statement-correctness test through a verifiable independent proof, although the route printed in this manuscript is incomplete. Because Zhang's paper appeared after the novelty cutoff, it is used here only to verify truth and not as evidence of non-novelty.

Verifiable source: Xin Zhang, Zaremba's conjecture for all denominators

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The composite-modulus mixing formulas are incorrect as written and leave their downstream branches incomplete. Independently, Lemma 32 and Proposition 35 contain unresolved obligations affecting the constructions used for Theorems 6–7. A separate false intermediate estimate in Proposition 35 has a verified replacement. Four literal notation defects are reported in yellow.

Lemmas 14–15 and Corollary 16: The composite-modulus mixing estimates have the wrong main term

Pages 9–10 · Equations (32)–(34) · arXiv:2603.14116v2

Incorrect as written

Take $q = 2^{Lm}$, $N = 2^m$ with a fixed sufficiently large L, and $A = B = \mathbb{Z}/q\mathbb{Z}$. For each c, the congruence $(a+2c)(b+2c) \equiv 1 \pmod q$ has exactly $\varphi(q) = q/2$ pairs, so Lemma 14's exact count is $Nq/2$, whereas its main term is Nq and its error is $O(qN^{1-\kappa})$. The discrepancy dominates that error. With $g(x) = x^{-1}$ and $S = [N]^2$, Lemma 15 has the same missing unit density: the discrepancy is $qN^2/2$, while its error is $O(qN^{2-2\kappa})$. For Corollary 16, let $N \mid q$, $I = [N]$, and $\Lambda = \{0, N, \ldots, q-N\}$, so $I \dotplus \Lambda = \mathbb{Z}/q\mathbb{Z}$; then the exact inverse intersection has size $\varphi(q) = q/2$, not the printed main term q. Immediate-consequence closure cannot change this density. Repair classification: No repair supplied; a corrected local density must be propagated through Lemmas 17–20 and the composite branches of the main proofs.

Verifiable source: Xin Zhang, corrected composite-modulus formulation

Lemma 32: Equation (107) drops an uncontrolled term

Pages 25–27 · Equations (91), (107), and (109)–(110) · arXiv:2603.14116v2

Incomplete as written

Write $x = x_{s_1}$ and $X = x_{s_* + \tilde{s}_*}$. The two inequalities in (91), together with $\mathcal{D}_2 \le D$ and $|\gamma_X| \le C_3$, give only $|\gamma_X^*| \le (C_3/|\gamma_{s_1}|)(y/x + DC_3 x/X)$. Equation (107) retains only a multiple of the second term. That requires an additional estimate comparable to $yX \le 3DC_3x^2$, which is neither assumed nor proved. Equations (109)–(110) use the reduced bound decisively. Repair classification: No repair supplied; the omitted y/x term must be controlled or the contradiction reorganized.

Proposition 35: Lemma 32's required coefficient bounds are chosen after its output

Pages 29–30 · Invocation of Lemma 32 through Equations (123)–(128) · arXiv:2603.14116v2

Incomplete as written

Lemma 32 requires a fixed vector $\vec{C} = (C_1, C_2, C_3)$ before producing a, b, c, x, y, z. Proposition 35 invokes the lemma first and then defines C_1, C_2, C_3 from X_1, X_2, X_3, which themselves depend on the resulting denominators and points. A statement of the form "for every fixed $\vec{C}$, there exists an output" does not supply an output for coefficient bounds chosen from that same output. Immediate-consequence closure cannot exchange these quantifiers. The proof also replaces the returned denominator y_0 by a later y without proving that the required independence is preserved. Repair classification: No repair supplied; a uniform, discretized, or simultaneous-selection argument is required.

Proposition 35, Equation (121): The printed criticality estimate is unavailable, but the needed bound is repairable

Page 29 · Equation (121) · arXiv:2603.14116v2

Incorrect as written · verified repair

The displayed estimate $|ax| \le q/(\widetilde{M}x_{s_0})$ would require x_{s_0} to be $\widetilde{M}$-critical, whereas it is selected only as an ordinary critical denominator. The conclusion needed in (121) nevertheless has a verified replacement. A convergent denominator gives $|ax| \le q/x$. From (120), $|I_*| \ge \delta^{60}|I|/5$, $x \ge \sqrt{Nq}/H$, $H = N^{9/20}$, and $\widetilde{M} \le N^{1/1000}$, one obtains $|I_*|x^2/q \ge 4N^{1/5}/\widetilde{M} > 1$. Hence $q/x \le |I_*|x$ and $|bx| \le |ax| + |b-a|x \le 2|I_*|x$. This repairs the required final bound but not the other independent gaps in Proposition 35.

Proposition 35 statement: The conclusion names a denominator instead of a partial quotient

Page 29 · Sentence following Equation (120) · arXiv:2603.14116v2

Typo

The proposition prints that for some denominator $q_j(a) \in [\sqrt{qN}/H, \sqrt{qN}]$ one has $q_j(a) \ge M_*$. Replace the latter assertion by: the next partial quotient associated with such a denominator satisfies $c_{j+1}(a) > M_*$. The proof immediately assumes the negation, namely that all partial quotients attached to denominators in this range are bounded by M_*. This uniquely fixes the object named in the conclusion and changes no intended argument.

Lemma 32 notation: The points attached to y and z are misidentified

Page 25 · Paragraph following Equation (103) · arXiv:2603.14116v2

Typo

The text prints $y = y(a), z = z(b)$. Replace it by $y = y(b), z = z(c)$. Lemma 32's statement and the immediately preceding selection $a < b < c$ determine the correction uniquely.

Proposition 35 notation: The convergent has the wrong argument

Page 30 · Paragraph following Equation (128) · arXiv:2603.14116v2

Typo

The text prints $x = q_s(s)$. Replace it by $x = q_s(a)$. The subscript s is the convergent index and a is the rational whose convergent is used, so the correction is mechanical.

Theorem 7, Equation (19): A stray multiplication symbol should be deleted

Page 5 · Equation (19) · arXiv:2603.14116v2

Typo

The lower bound is printed as $q^{2w_M-1-o(1)}\cdot$ with no following factor. Delete the trailing $\cdot$. The intended expression is $q^{2w_M-1-o(1)}$, and no later calculation contains an additional factor at this point.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2603.14116v2
2. Xin Zhang, corrected composite-modulus main term
3. Xin Zhang, corrected composite-modulus formulation
4. Bourgain-Kontorovich, density-one denominator result
5. Aistleitner-Borda-Hauke, earlier partial-quotient bounds
6. Moshchevitin-Murphy-Shkredov, earlier maximum bound
7. Shulga, radical-dependent denominator bound

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.