MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On the generating series of the degree sequence

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Prompt	Standardized prompt
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Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The natural-boundary criterion for monomial maps, its two stated corollaries, the sharp surface dichotomy including transcendence after reduction modulo sufficiently large primes, and the rigidity theorem for equal degree sequences are correct. The printed proofs contain several precisely local defects, but each has a verified repair and none changes a central conclusion.

Theorem A: Natural boundary under a unique dominant conjugate pair

Pages 2 and 9–12 · Theorem A and Section 4.1 · arXiv:2510.06142v2

Correct

The mixed-volume formula expresses the degree sequence as $h(\Lambda^k A^n)$ for a piecewise-linear function h. The spectral gaps isolate the two-dimensional real space on which $\Lambda^k A$ acts as multiplication by $\lambda_k e^{i\theta}$, with $\theta/(2\pi)$ irrational, while the complementary contribution has larger convergence radius. Proposition 3.2 and Skolem–Mahler–Lech give an absolutely summable partial-fraction expansion with nonzero residues on an arithmetic progression; its pole set is dense on $|z| = \lambda_k^{-1}$. Replacing the overly sharp complementary estimate and adding distinctness to Lemma 4.5, as verified below, completes the proof and establishes the claimed natural boundary, transcendence, and non-holonomicity.

Verifiable source: Full paper, version 2

Corollaries B and C: Bell–Diller–Jonsson application and infinite algebraic independence

Pages 3–4 and 12–13 · Corollaries B–C and Section 4.2 · arXiv:2510.06142v2

Correct

The identity cited for the Bell–Diller–Jonsson construction separates the pole at $\lambda_1(f_\mu)^{-1}$ from the monomial series whose boundary is $|z| = \lambda_2(f_\mu)^{-1/2}$, giving Corollary B. For Corollary C, iterates of one admissible monomial map have pairwise distinct k-dynamical degrees and hence pairwise distinct natural-boundary radii. Lemma 4.7 is valid after choosing a minimal squarefree algebraic relation: in characteristic zero its discriminant is nonzero, so the holomorphic implicit-function theorem would continue the series having the smallest radius across a boundary point, a contradiction.

Verifiable source: Full paper, version 2

Theorem D: Surface dichotomy and transcendence modulo large primes

Pages 4 and 13–17 · Theorem D and Sections 5.1–5.2 · arXiv:2510.06142v2

Correct

In the irrational-angle conjugate case, the repaired endpoint form of Theorem A gives the natural boundary. In every remaining eigenvalue case, Favre's stabilization theorem and Cayley–Hamilton give a linear recurrence and hence a rational generating series. For the reduction claim, Christol's theorem reduces algebraicity to weak periodicity. Lemma 5.5 supplies a non-linear convex integral piecewise-linear function, and an irrational rotation necessarily moves some point of any proper coefficient arc outside that arc; after excluding finitely many primes, the corresponding coefficient values remain distinct in the residue field. This contradicts weak periodicity. The citation-scope and arc-selection defects in the printed proof have the verified repairs recorded below.

Verifiable source: Favre, Les applications monomiales en deux dimensions

Theorem E: Rigidity from equality of degree sequences

Pages 4–5 and 17–19 · Theorem E and Section 5.3 · arXiv:2510.06142v2

Correct

In the real-eigenvalue case, equality of dynamical degrees and quadratic Galois conjugacy give equal eigenvalue multisets for A^2 and A'^2, hence rational conjugacy and an integral semiconjugacy. In the irrational-angle case, the corrected radial-residue formulas identify a rank-one lattice of coincident poles. Equality of the residue recurrence sequences forces the two generator coordinates to have equal absolute value; otherwise Equation (5) equates a nonzero power sum of degree at least two with one of degree at most one. The resulting root-of-unity relation and the quadratic-field argument exclude orders $5, 8, 10, 12$, leaving exactly $u \in \{1, 2, 3, 4, 6\}$.

Verifiable source: Full paper, version 2

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The central conclusions admit verified repairs, but the printed proof is not fully correct as written. The complementary spectral estimate ignores possible Jordan growth; Lemma 4.5 is false without distinct pole parameters; the rational branch of Theorem D invokes a result whose stated scope is only birational maps of $\mathbb{P}^2$; and the modulo-p proof's two-case arc selection is not valid for all rotations. Boundary notation and the radial-limit arguments also require local corrections.

Lemma 4.2 and Step 2 of Theorem A: The complementary term need not be $O(\lambda^n)$

Pages 10–11 · Lemma 4.2 and the display immediately following it · arXiv:2510.06142v2

Incorrect as written · verified repair

The proof defines $\lambda = \lambda_{k-1}|\mu_{k+2}|$ as the largest possible modulus on the complementary invariant space and then asserts an $O(\lambda^n)$ bound. A Jordan block at an eigenvalue of modulus λ invalidates this estimate. For an explicit admissible example, take $d = 5$, $k = 2$, and

$$A = \operatorname{diag}\left(10, \begin{pmatrix} 0 & -2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\right).$$

Its ordered eigenvalue moduli are $10, \sqrt{2}, \sqrt{2}, 1, 1$; the conjugate-pair ratio is not a root of unity, while $\Lambda^2 A$ has a Jordan block at eigenvalue $10 = \lambda$, so its powers grow like $n10^n$. Repair classification: Verified repair. Choose any $\widetilde{\lambda}$ with $\lambda < \widetilde{\lambda} < \lambda_k$. Jordan normal form gives $O(\widetilde{\lambda}^n)$ on the complement, which still makes the remainder analytic in a disk strictly larger than $|z| < \lambda_k^{-1}$ and leaves every later step unchanged.

Lemma 4.5: The residue formula requires pairwise distinct pole parameters

Pages 11–12 · Lemma 4.5 and its proof · arXiv:2510.06142v2

Incorrect as written · verified repair

The lemma allows repetitions among the points p_m but claims $\lim_{\rho\to 1^-}(1-\rho)\Delta(p_m^{-1}\rho) = b_m$. If $p_0 = p_1 = 1$, $b_0 = 1$, $b_1 = -1$, and every other $b_m = 0$, then $\Delta = 0$ and the claimed limit for $m = 0$ is $0 \neq b_0$. In general the limit is the sum of all coefficients attached to that same point. Repair classification: Verified repair. Add the hypothesis that the p_m are pairwise distinct. In the only application, $p_m = e^{im\theta}\lambda_k$ and $\theta/(2\pi)$ is irrational, so distinctness holds and the dominated-convergence proof is valid without any downstream change.

Theorem D · rational branch: Diller–Favre Corollary 2.2 is invoked outside its stated scope

Page 14 · second case in Section 5.1 · arXiv:2510.06142v2

Incorrect as written · verified repair

The paper applies Diller–Favre, Corollary 2.2, to an arbitrary dominant monomial self-map of a projective toric surface. That corollary is stated for birational self-maps of $\mathbb{P}^2$, whereas here $|\det A|$ may exceed 1 and the toric surface need not be $\mathbb{P}^2$. Repair classification: Verified repair. Favre's preceding result supplies a toric birational model on which the lift is 1-stable. On that model, the degree is a fixed linear functional of $((\widetilde{\varphi})^*)^n$ applied to the pulled-back divisor. Cayley–Hamilton therefore gives a constant-coefficient recurrence, and projection formula identifies it with the original degree sequence. Hence the generating series is rational in the claimed remaining cases.

Verifiable source: Diller–Favre, Corollary 2.2

Theorem D · reduction modulo p: The printed arc selection does not follow from the phase shift

Pages 16–17 · the two bullets following Equation (4) · arXiv:2510.06142v2

Incorrect as written · verified repair

Writing $s = \{(q-1)(r-r')\theta/(2\pi)\}$, the phases satisfy $\{r_n\theta/(2\pi)\} = \{r'_n\theta/(2\pi) + s\}$. The two printed choices near the left endpoint of $I_1 = [\alpha, \beta]$ need not put one phase in I_1 and the other outside; for example $I_1 = [0.3, 0.7]$ and $s = 0.1$ contradict the second bullet's proposed configuration. Repair classification: Verified repair. Since $0 < s < 1$, rotation by s cannot preserve the proper closed arc I_1. Choose $x \in I_1$ with $x + s \notin I_1$, then use density of the r'_n phases to place one sufficiently close to x. The same distinct-residue calculation gives the desired contradiction. In defining the finite bad set, any zero coefficient c_i should simply be omitted, and one may also include μ_2 (or $\det A$) among the finitely many elements required to remain units. These local changes establish the printed conclusion.

Theorem A · endpoint hypotheses: The extreme values of k use nonexistent neighboring eigenvalues

Page 2 · first hypothesis of Theorem A, propagated to Step 2 on page 10 · arXiv:2510.06142v2

Minor formal correction

The theorem permits $1 \le k \le d-1$ but writes $|\mu_{k-1}| > |\mu_k| = |\mu_{k+1}| > |\mu_{k+2}|$. Thus μ_0 occurs when $k = 1$ and μ_{d+1} occurs when $k = d-1$; for $d = 2, k = 1$, both boundary defects occur, even though Theorem D later uses this case. Read each unavailable outer inequality as vacuous. In Step 2, define λ as the spectral radius of the remaining exterior-power eigenvalues; when the complementary space is zero, take the remainder $H = 0$. The same strict spectral gap gives $\lambda < \lambda_k$. This is a consistent local endpoint repair and changes no result.

Theorem E · radial residue formulas: The radius factor is omitted or inverted in three arguments

Page 18 · first three radial-limit displays in the complex-eigenvalue case · arXiv:2510.06142v2

Typo

The partial fractions have denominators $1 - e^{im\theta}\lambda_1 z$, so Lemma 4.5 must be evaluated at

$$z = \rho e^{-im\theta}\lambda_1^{-1}.$$

The two matched-pole limits omit λ_1^{-1}, while the following unmatched-pole limit prints λ_1 instead of λ_1^{-1}. Insert λ_1^{-1} in all three places. The intended correction is uniquely determined by the displayed partial fractions and restores the residue comparison without changing the argument.

Remaining central proof chain: Mixed-volume, Fourier, algebraic-independence, and rigidity steps

Pages 7–19 · Propositions 3.2 and 4.1, Lemmas 4.3–4.4 and 4.7, Lemmas 5.5–5.8, and Section 5.3 · arXiv:2510.06142v2

Correct and complete

The mixed-volume decomposition produces a finite piecewise-linear degree functional; its restriction to the conformal plane has absolutely summable Fourier coefficients. Skolem–Mahler–Lech supplies the nonzero arithmetic progression needed for density. Lemma 4.7 follows after the standard minimal squarefree normalization of the assumed relation. The convexity and integral-coefficient properties in Lemma 5.5, the finite-intersection argument in Lemma 5.8, and the power-sum degree comparison in Equation (5) are valid. With the explicit local repairs above, no further nontrivial case or dependency remains missing.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 2
2. Favre, Les applications monomiales en deux dimensions
3. Diller–Favre, Corollary 2.2

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.